Lecture 1 – Scales: Familiar and Unfamiliar

Physics for Pedestrians

15th July, 2019

1 Scientific Notation

In physics we often have to deal with numbers that are uncomfortably large, and perform operations on them. This will not be possible without scientific notation. Consider the speed of light, usually denoted by the letter c. The speed of light is the number of metres (a length) that a photon of light covers in one second. Experimentally, this number has been found to be:

$$c = 300,000,000 \text{ m/s}$$

Such numbers are very large, and as a result it is very easy to make mistakes when we manipulate them. Luckily, there is a way out of this: we could try to write the above number in the following way:

$$c = 3 \times 10^8 \text{ m/s}$$

We have done two things by writing it like this:

1. We have collected all the zeros together (all 8 of them) and represented them as 10^8 (this is basically shorthand for saying that it is $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$).
2. The remaining information has been included as a number between 1 and 10 (in this case 3).

Let us now slightly generalise this. We write any number as:

$$\text{Any number} = N \times 10^x \tag{1}$$

where N lies between 1 and 10, and is called the *coefficient*, and x is called the *exponent*.

Now this method can be used just as well for numbers that are less than 1. For example, consider the number 0.0005. This number is just[1]

$$0.0005 = \frac{5}{10000} = \frac{5}{10^4} = 5 \times \frac{1}{10^4} = 5 \times 10^{-4}.$$

So writing a number in "scientific notation" is a two-step process:

[1] If you're having trouble with negative powers, take a look at this video, or if you prefer, read through this site.

1. Locate the decimal point,[2] and move it to the right or left so that there is only one non-zero digit to the left of the decimal point.

   In the above example, we move the decimal point to the right 4 times, until we are left with the number 5.0. This gives us N in Equation (1).

2. Count the number of spaces you had to jump. This is the exponent x. If you jump to the left, **add 1 to** x, but if you jump to the right, **subtract 1 from** x.

   In the above example, we jumped to the **right** 4 times, so $x = -4$.

Here are some examples:

- **Number = 0.01:** Jump to the **right** **two** times to get

$$1.0 \times 10^{-2}$$

- **Number = 580.11:** Jump to the **left** **two** times to get

$$5.8011 \times 10^{+2}$$

- **Number = 0.00038:** Jump to the **right** **four** times to get

$$3.8 \times 10^{-4}$$

- **Number = 690000:** Jump to the **left** **five** times to get

$$6.9 \times 10^{+5}$$

This is useful for many reasons. Such numbers are very easy to manipulate, since the powers of ten simply add. Here's an example:

$$\underbrace{1000000}_{10^6} \times \underbrace{100000}_{10^5} = \underbrace{100000000000}_{10^{11}}$$

$$\underbrace{0.0001}_{10^{-4}} \times \underbrace{100}_{10^2} = \underbrace{0.01}_{10^{-2}}$$

This makes life very easy. Suppose for example you wanted to calculate the number of seconds in a day. This is how you would do it:[3]

[2] If the number doesn't have one, then it's just after the last digit, for example 300000000 is shorthand for 300000000.00. The decimal point is usually ignored in this case as it is redundant

[3] **Important:** If you don't understand this example, go and read **Appendix A** at the end of this document right now and come back.

$$
\begin{aligned}
\text{Number of seconds in a day} &= 1 \text{ day} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{60 \text{ seconds}}{\text{minute}} \\
&= 1 \cancel{\text{day}} \times \frac{24 \cancel{\text{hours}}}{\cancel{\text{day}}} \times \frac{60 \cancel{\text{minutes}}}{\cancel{\text{hour}}} \times \frac{60 \text{ seconds}}{\cancel{\text{minute}}} \\
&= 1 \times 24 \times 60 \times 60 \text{ seconds} \\
&= 1 \times 2.4 \times 10^1 \times 6 \times 10^1 \times 6 \times 10^1 \text{ seconds} \\
&= 86.4 \times 10^3 \text{ seconds} \\
&\approx 10^5
\end{aligned} \tag{2}
$$

1.1 "Equalities"

You might notice that the last "equals" sign above is a little squiggly. There's a special reason for this: clearly

$$86.4 \times 10^3 = 86,400 \neq 100,000$$

The squiggly equals $A \approx B$ means that A is **approximately equal to** B.[4]

Here is a collection of different "equalities" that we'll use during this course:

Symbol	**Meaning**	**To be read as**
$=$	Equality	"is equal to"
$\approx$	Equality except for a purely numerical factor near 1	"is approximately equal to"
$\sim$	Equality except for a purely numerical factor	"is roughly equal to"
$\propto$	Equality except for a factor that may have dimensions	"is proportional to"

As usual, here is an example of how to use these signs:

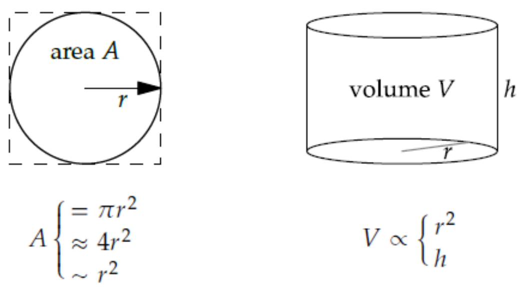

Figure 1: *On the left:* The area of a circle is exactly $A = \pi r^2$. But since $\pi = 3.14 \approx 4$, $A \approx 4r^2$. Ignoring the numerical factor, we can say $A \sim r^2$. *On the right:* The volume of the cylinder depends both on its radius-squared and height, and so is proportional to both quantities independently.

[4]This shouldn't be too hard for you to make sense. When asked how far away Delhi is from Ashoka, you'd probably say "roughly" 30 km. Such vague questions deserve appropriately vague answers!

Question: Show that if you counted the number of seconds in 100 days, you'd get:

$$100 \text{ days (in seconds)} = 10^7 s$$

Question: Show that 100 years $\approx 10^9 s$.

1.2 Why approximate?

A natural question you might be asking yourself is the following: why is an approximate value useful in any way? However, this should not be too surprising to you, since it's something you do quite regularly: when asked how far Ashoka is from Delhi, you may say something like "Oh, roughly 20 or 30 kilometres".[5] This is far from a definite response, and yet it causes you no discomfort. One of the reasons for this is quite simple: the question "How far away is Delhi" is extremely vague. What does "Delhi" even mean? The border? The airport? The centre of the city? Being too precise is a waste of time; the person asking the question would hardly require the distance from Ashoka to the Qutub Minar precise to the nearest metre, as that it not the *spirit* in which the question was asked.

The same is true in the sciences. Very often, scientists need to make "order-of-magnitude" estimates, or rough calculations, which will allow them to decide if a problem is indeed worth pursuing. Depending on the result of their calculation, they may decide to devote some of their time to solving the problem, or move on to a different question.

1.3 Prefixes

Sometimes, a quantity is too small for it to be used regularly. In everyday life, imagine you had to work with paise instead of rupees. This would be extremely annoying and you'd probably find it redunant (a coffee at *Chai-Shai* would be 5000 paise, the cost of a taxi to the airport would be 1,00,000 paise, ...). Since nothing we buy everyday is of the order of a paise, we use the rupee, which is significantly more convenient.

Similarly, some physical quantities are defined using standard values that are very small or large.[6] To deal with this, we introduce **prefixes** which can be used as a shorthand. For simplicity, these prefixes are usually related to powers of 10. These are the ones we will use:

Power of 10	Prefix	Meaning
10^{+6}	mega-`unit`	$10^{+6}\times$ `unit`
10^{+3}	kilo-`unit`	$10^{+3}\times$ `unit`
10^{0}	`unit`	$1\times$ `unit`
10^{-3}	milli-`unit`	$10^{-3}\times$ `unit`
10^{-6}	micro-`unit`	$10^{-6}\times$ `unit`

The `unit` here could be anything: metres, grams, tonnes, etc.

[5] In fact, what's far more likely is that you will say, roughly an hour's drive from Delhi. But this is even more imprecise, as it assumes the average speed that the shuttle takes! In fact, if you think Delhi is an hour away by shuttle, you're in for a rude surprise!

[6] For example, a gram of mass is a very small unit and almost nothing you buy will be of that order.

2 Desert Island Measurements: the scale of the familiar

One of the possible titles of this course was *Desert Island Physics*. Imagine you're stuck on a desert island with no tools except your wits, and I ask you what kinds of physical quantities you can measure with only your senses (you have no instruments with you!).

Three different quantities that you can measure are **masses**, **distances (or lengths)**, and **time intervals**. A valid question, as pointed out in class, is what about other quantities like temperature or speeds. It turns out (we'll see in a later class) that most quantities – and certainly all quantities we're interested in this course – can be written as a combination of masses, lengths, and time intervals.

There's nothing *strictly* stopping you from defining quantities like temperature or speed as "fundamental" and deriving masses, lengths, and times from them, but this turns out to be very difficult and frankly unnecessary: mass, length, and time are very simple to understand physically, and easy to manipulate.

Quantities which can be measured are said to have **dimensions**. This is the difference between the number 3 and the length 3m. When measured in cm, 3m = 300cm. In other words, its numerical value changes. This is a characteristic of an object with what physicists call "dimension".

A dimensionless number is one that doesn't change when we change units. For example, suppose you're on a diet and you want to know how much of your mass you've lost. After a month, you see that your mass has dropped from 100 kg to 75 kg. You could say[a] to everyone that you have **lost** one-fourth of your mass, since:

$$\text{Fraction lost} = \frac{\text{Mass lost}}{\text{Original mass}} = \frac{100\text{kg} - 75\text{kg}}{100\text{kg}} = \frac{25\text{kg}}{100\text{kg}} = \frac{1}{4}.$$

This fraction is *dimensionless*, in other words, if you measured mass in grams, then you were 100,000 grams and your mass dropped to 75,000 grams, but the fraction of your mass lost *still* remains one-fourth!

These are *not* to be confused with the "dimensions" of space. The colloquial definition of the word dimension causes a lot of confusion here. For the purposes of this course, when we say dimensions, we mean **physical quantities like mass, length, and time which can be measured**.

[a] proudly!

As these objects can be measured, they are usually measured according to some **standard**. This standard is known as a **unit** of measurement. For example, a length can be measured in units of feet or metres, for example; time can be measured in units of seconds, minutes, or hours; masses can measured in kilograms, pounds, or (if you're feeling unkind) tonnes.

Question: Which of the following are dimensions, and which are units?

`mass, kilogram, minute, light-year, second, length, time, metres,`
`(a little harder) force, power, joule, newton, energy`

So let's try to ask the following vague question: what are the smallest and largest masses, lengths, and times that we can measure with our bare senses, using no other instruments.

2.1 Masses

You rummage through your pocket, and you find a coin, a board-pin, a paperclip, and a grain of rice. You close your eyes and try to weigh them with your hands.[7]

You'll notice that it's easy to tell apart the coin and the grain from the other two (and from each other) – the coin is much heavier and the grain much lighter. But if you were to compare the board-pin and the paperclip, you'd find it hard to see which was heavier. You estimate that they must both be about a gram (give or take a little bit), and so you can – at best – measure mass up to a fraction of a gram, say 0.1 g.

Question: Show that $0.1\text{ g} = 10^{-4}\text{ kg}$:
Hint: Begin by writing:

$$0.1\text{ g (in kg)} = 0.1\text{ g} \times \frac{1\text{ kg}}{1000\text{ g}}$$

since that is the definition of the "kilo-"gram.

What about the largest mass you could measure? Beyond a point, you'd find it hard to carry something. Since you remember being able to carry a person, whose weight you estimate to be of the order of 100 kg, you decide to play it safe and say that you can tell the difference between a 100 kg and a 1000 kg, but not beyond that.[8]

2.2 Lengths

What about lengths? The smallest length you can possibly measure with your bare senses is roughly of the order of the thickness of a hair. You can certainly tell two hairs apart, and so you decide that the lower limit is around a fraction of a millimetre (mm), say 0.1 mm.

Question: As before, Show that $0.1\text{ mm} = 10^{-4}\text{ m}$:

The largest length is a little harder. Without any points of reference (a completely clear ocean with no sign of rescuing ships, for example) you'd be hard-pressed to tell the difference between a

[7] It's sad that there is no way that the word "mass" can be used as a transitive verb in the same way that "weigh" can, e.g. This coin *masses* more than the board-pin.
[8] In fact, given a lump of some rock, you'd be hard-pressed to say how much it weighed without knowing what it's made of, i.e. its density.

kilometre and 10 kilometres. The horizon (seen from a tall hill) would be around 10 km, so let's give ourselves the benefit of the doubt and say that this is the maximum length we can measure with our senses.

2.3 Time-intervals

Intervals of time are a little trickier: you can measure your pulse, which is of the order of a second, and can tell when your heart is racing faster than usual. The smallest interval is about a blink of an eye. This is about a fraction of a second, say 0.1 s $= 10^-1$ s.[9]

The longest intervals of time you can accurately measure are also quite tricky to determine, especially without recourse to the seasons and the sun or moon. You could certainly tell the difference between a day and a month, but you'd probably find it hard to tell the difference between a month and a year. Let's be safe and say 100 days.

Question: Show that 100 days (in seconds) $= 10^7$ s:
Hint: Begin by writing:

$$100 \text{ days (in s)} = 100 \text{ days} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}}$$

We can now depict this on a graph, as done in Figure (2). It should strike you, however, that the fact that we have chosen kg, m, and s as the units of measurement is a result of the scale of our personal lives. A beetle or an elephant might find these units quite badly chosen (just as we no longer find the paisa a useful unit of money, while our grandparents certainly did).

For the enthusiastic reader

The fact that the units for mass and length lie symmetrically about the range of our perception is a reflection of the fact that they are "natural" units in which we work every day. When asked the size of a room, you'd give the answer in metres (or feet, which is the same order of magnitude); the carry on baggage weight restrictions on airplanes are given in kilograms, etc. The fact that the second is not symmetrically placed in between the smallest and largest times we can perceive is perhaps because it is *not* the natural unit of time in which we live our everyday lives (you'd hardly say a trip to Delhi took 4,800 seconds). This can be easily remedied, if you're interested, by choosing a new unit of measure (say, one hour, or 30 minutes) and performing the calculation again. If you do this, you'll see that our perception of time lies symmetrically about this new unit.

[9] You can test this: using your pulse as a clock, blink your eyes repeatedly and see how many blinks you complete in a second. You should find that it's roughly 2 or 3 blinks per second.

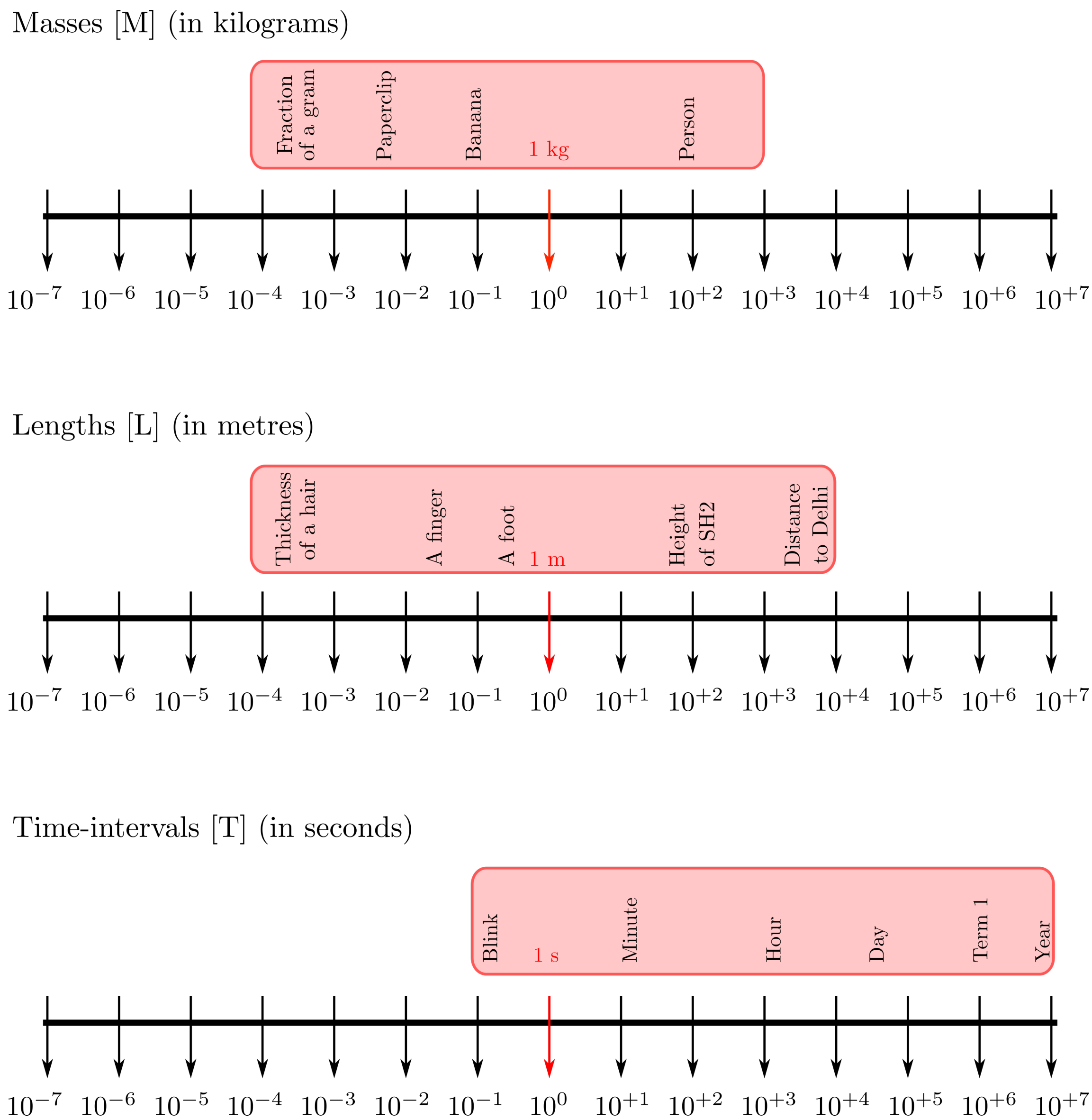

Figure 2: The masses, lengths, and time-intervals we can perceive with our bare senses. The units in each case are the standard international units of kg, m, and s respectively. Each interval marks a power of 10, in other words, moving to the right, each interval is 10 times larger than the one before it. For example, a blink is 10 times smaller than a second, which is 10 times smaller than a minute, and so on.

3 Laboratory Measurements: the scale of the unfamiliar

Let us now leave the desert island[10] and move to the laboratory where scientists have been working tirelessly to stretch the limits of the smallest and largest units that can be measured (either theoretically or experimentally).

> If you don't completely understand this part, relax! What you need to take away is that theoretical and experimental predictions have given us the scales of masses, lengths, and times of the universe, and they are astronomically larger (and smaller!) than the scales of our perception.

Just as kg, m, and s seem to be natural units for us, Nature has certain quantities which it uses to define "natural" units for itself. These are certain "constants" of nature, Newton's Gravitational constant (G), the speed of light (c), and Planck's constant (h). It doesn't matter if you don't know what they are, just that these are numbers with dimension (i.e. they are measurable in units of kg, m, and s). From these numbers we can *construct* natural units. (Again, don't worry about *how* this can be done, we'll see that in due course).

According to our current understanding of the world, there is a theoretical limit to the smallest length and time below which it does not make sense to speak of lengths or times. Below these lengths and times, our theories break down: as we make progress, perhaps these barriers will be lifted. These limits are known as the *Planck length* ($l_p = 10^{-35}$ m) and the *Planck time* ($t_p = 10^{-35}$ s) respectively. There seems to be no theoretical lower limit to the mass of a particle (some subatomic particles are known to have zero mass). Experimentally, we have been able to measure masses as small as that of an electron, m_e, roughly 10^{-30} kg.

For the largest times, lengths, and masses, we turn to the scale of the known universe. The age of the universe has been established to be roughly 13.7 billion years,[11] so this is a good estimate of the time scale of the universe.

> **Question:** What is 13.7 billion years in seconds?
> **Hint:** "Billion" is shorthand for 10^9.

The size of the visible universe is again difficult to establish. A simple way to go about it is to imagine that the furthest object out there which we can see is at the edge of the universe, and that the light we are seeing was emitted as soon as the universe was born. In that case, this object is a distance of (speed of light $\times$ age of universe) away, roughly 10^{25}m.

The mass of the visible universe is also quite hard to establish, but one way would be to count the number of galaxies, estimate the number of stars in a galaxy, and then multiply that by the mass of an average star. There are roughly 10^{11} galaxies, with 10^{11} stars each. If we assume each star to be the mass of our sun (about 10^{30} kg), this leaves us with a total mass of 10^{50} kg.[12] Let's now represent this as before, in Figure (3).

[10] You make a raft, wait for the right time of the year, and set sail. You lose Wilson, and are finally found by a passing cargo ship. You try to sell the film rights to Hollywood, but find someone has beaten you to it.

[11] This can be done by looking at the expansion of the universe and extrapolating backwards, or by finding the ages of different stars.

[12] You could do this another way: the density of the universe is reasonably well known through satellite data, and thought to be roughly 10^{-30}g/cc. Imagine the universe to be a sphere of that density and calculate its mass.

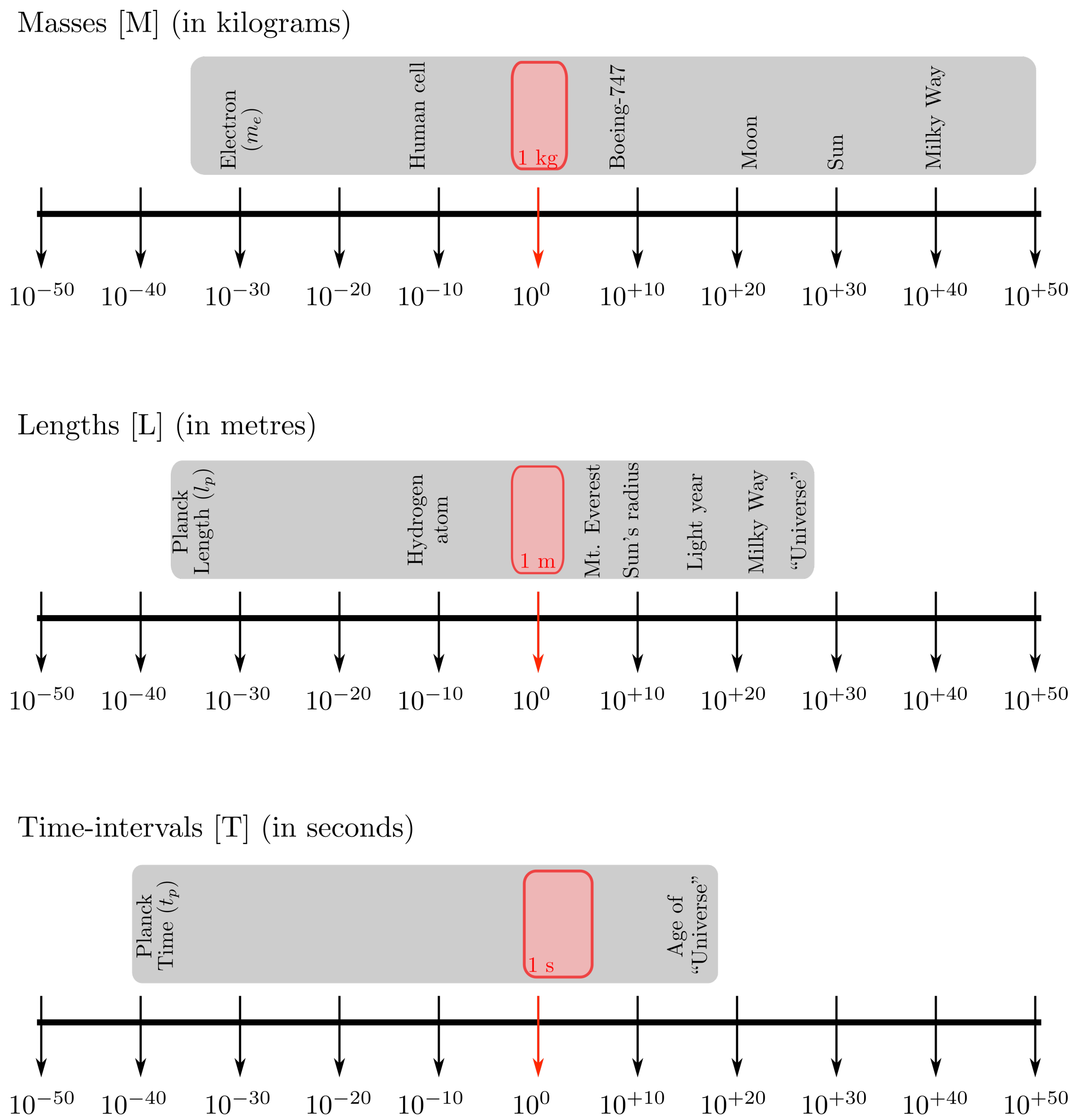

Figure 3: The masses, lengths, and time-intervals of the universe with our current understanding. The scales we perceive with our senses make up a tiny part. Each division is a factor of 10,000,000,000: the moon is 10,000,000,000 times less massive than the sun, etc. The point? **The universe is essentially invisible to our senses.**

If you are not awed by the scale of this figure, it is probably because you're misunderstanding powers of 10. Remember: your height and the height of the Admin block differ by a factor of 10. The Admin block and Mt. Everest differ by a factor of 100. $10 \times 100 = 10^3$ has already taken you a fair distance. Now, 10^{10} is the difference between your height and a single Hydrogen atom's.

Appendices

A Converting units

In the next class we will spend a lot of time converting units, so it's good if you get the hang of it soon. Suppose you are asked to write 1 gram in kilograms. You know that

$$1 \text{ kg} = 1000 \text{ g}$$

From this, it should be clear to you that

$$\frac{1 \text{ kg}}{1000 \text{ g}} = 1$$

So you being by writing:

$$\begin{aligned}
1 \text{ g (in kg)} &= 1 \text{ g} \times 1 \\
&= 1 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} \text{ (from the earlier equation)} \\
&= 1 \not{\text{g}} \times \frac{1 \text{ kg}}{1000 \not{\text{g}}} = \frac{1}{1000} \text{ kg} \\
&= 0.001 \text{ kg}
\end{aligned}$$

This should be painfully simple. Let's try a slightly harder question. What is 1 pound in grams?

Solution: We know that

$$\begin{aligned}
1 \text{ kg} &= 2.2 \text{ pounds, and} \\
1 \text{ kg} &= 1000 \text{ g}
\end{aligned}$$

Re-arranging as before, we have

$$\frac{1 \text{ kg}}{2.2 \text{ pounds}} = 1 = \frac{2.2 \text{ pounds}}{1 \text{ kg}}$$

and, as we saw earlier,

$$\frac{1 \text{ kg}}{1000 \text{ g}} = 1 = \frac{1000 \text{ g}}{1 \text{ kg}}$$

$$\begin{aligned}
1 \text{ pound (in grams)} &= 1 \not{\text{pound}} \times \frac{1 \not{\text{kg}}}{2.2 \not{\text{pounds}}} \times \frac{1000 \text{ g}}{1 \not{\text{kg}}} \\
&= \frac{1000}{2.2} \text{ g} \approx 454 \text{ g}
\end{aligned}$$

Question: You'll notice that in the second example we used

$$\frac{1000\ \text{g}}{1\ \text{kg}} \text{ instead of } \frac{1\ \text{kg}}{1000\ \text{g}}$$

Why do you think that is the case?

Here's an even harder question: convert 1 km/h into m/s.

Solution: Start by writing the individual conversions between lengths and times. You know that

$$\begin{aligned} 1\ \text{km} &= 1000\ \text{m} \\ 1\ \text{hour} &= 60\ \text{min} \\ 1\ \text{min} &= 60\ \text{s} \end{aligned}$$

Then, write

$$\begin{aligned} 1\ \text{km/hr (in m/s)} &= \frac{1\ \text{km}}{1\ \text{hour}} \times \underbrace{\frac{1\ \text{hour}}{60\ \text{min}}}_{=1} \times \underbrace{\frac{1\ \text{min}}{60\ \text{s}}}_{=1} \times \underbrace{\frac{1000\ \text{m}}{1\ \text{km}}}_{=1} \\ &= \frac{1\ \cancel{\text{km}}}{1\ \cancel{\text{hour}}} \times \underbrace{\frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{min}}}}_{=1} \times \underbrace{\frac{1\ \cancel{\text{min}}}{60\ \text{s}}}_{=1} \times \underbrace{\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}}_{=1} \\ &= \frac{1000}{3600}\frac{\text{m}}{\text{s}} = \frac{5}{18}\frac{\text{m}}{\text{s}} \end{aligned}$$