Lecture 4 – The Clockwork Universe

Physics for Pedestrians

26th July, 2019

1 The Third Law: Conservation of Momentum

Newton's three laws provided a framework to describe the entire universe as it was then known. It explained the motions just as well as it explained the motion of celestial bodies, and the rules that they followed were simple.

Let us now look at a result of Newton's third law: forces come in equal and opposite pairs that act on *different* bodies. In order to understand this, imagine two masses m_1 and m_2 which are travelling on a straight line towards each other with velocities v_1 and v_2 respectively, as shown in Figure (1). These objects are not attracting each other, and – at this instant – there is no force between them. However, as they collide, there is a **contact** force which arises because of their interaction. During the collision, both bodies feel a force: m_1 exerts a force of F_{12} on m_2, and m_2 exerts a force of F_{21} on m_1. By the third law, we know that

$$F_{12} = -F_{12} \tag{1}$$

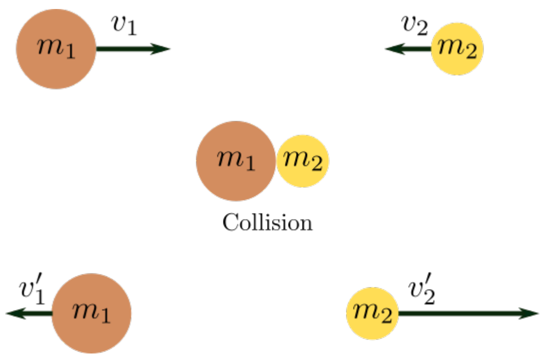


Figure 1: Two objects before, during, and after collision.

Remember, these forces act on *different* objects: F_{12} is the force *of* m_1 acting on m_2, and vice versa.

This would lead to a change in the velocities of these masses. Let us call these new velocities v_1' and v_2'. Where[1]

$$
\begin{aligned}
v_1' &= v_1 + \Delta v_1 \\
v_2' &= v_2 + \Delta v_2
\end{aligned}
\tag{2}
$$

But what are the forces F_{12} and F_{21} in terms of the masses and (changes in) velocities of the objects?

We know that the acceleration of an object is given by the change in its velocity in a small interval of time. Thus,

$$
a = \frac{\Delta v}{\Delta t} \tag{3}
$$

If we imagine that the two objects collide over some interval of time Δt, then it should be clear that the force F_{12} is given by

$$
F_{12} = m_1 \frac{\Delta v_1}{\Delta t} \tag{4}
$$

Similarly,

$$
F_{21} = m_2 \frac{\Delta v_2}{\Delta t} \tag{5}
$$

But from Newton's Third Law, we have that $F_{12} = -F_{21}$, and so

$$
m_1 \Delta v_1 = -m_2 \Delta v_2 \tag{6}
$$

Looking at this, we see something curious: it looks like that amount of this quantity $m \times v$ (known as **momentum**) which is lost (or gained) by the mass m_1 is gained (or lost) by the mass m_2. In other words, it looks like the the quantity $m_1 \times v_1 + m_2 \times v_2$ is constant or **conserved**, despite the collision. Let us try to prove this:

Let us say that the quantity[2]

$$
m_1 v_1 + m_2 v_2 = p \tag{7}
$$

Now let us look at the same quantity *after* the collision:

[1] The Δ notation denote a *change* in an object. Thus, Δv_1 would be the change in the velocity v_1, and this is how it should be read.

[2] The notation p for momentum comes from the Latin *petere*, which was the root word of a concept known as *impetus* which in turn was the predecessor of the concept of momentum.

$$
\begin{aligned}
m_1 v_1' + m_2 v_2' &= m_1(v_1 + \Delta v_1) + m_2(v_2 + \Delta v_2) \\
&= \underbrace{(m_1 v_1 + m_2 v_2)}_{=p} + \underbrace{(m_1 \Delta v_1 + m_2 \Delta v_2)}_{=0,\ \text{from Equation (6)}} \\
&= p
\end{aligned} \tag{8}
$$

Thus, this quantity p is the same before and after the collision. Thus, it is a **conserved quantity**.

> Let's examine why Newton might suggest we wear seatbelts: suppose you're sitting in a car (of mass 2000 kg) moving at some velocity (say, 20 km/hr).
>
> **Question:** What is the momentum of the system (both you and the car)? Now imagine that the car crashes against a brick wall, coming to rest.
>
> **Question:** What should **your** momentum be, so that the total momentum is conserved?
>
> **Question:** If you stopped sharply, this could kill you. What do you think the seatbelt is designed to do, in order to reduce the force on you? **Hint:** Look at Equation (3).

2 The Second Law: Accelerations and Forces

As described earlier, the second law establishes – in an inertial frame – the relationship between a force and a perceived acceleration. Our bodies are accelerometers, not speedometers. What this means is that we cannot have a sense of the speed with which we are travelling,[3] we can only sense accelerations. For example, when you're on an aeroplane that's travelling at a great velocity, it does not seem any different from travelling on a train or in a car, which move much slower in comparison. Indeed, it's so much like firm ground that the stewards serve you coffee and food, despite the fact that you're hurtling through the sky at a tremendous pace with respect to the Earth. However, the minute there is turbulence (which is essentially a *change* in velocity due to pockets turbulent air which exert a force on the aeroplane) you most certainly feel it!

But, you may protest, when you are on a motorcycle, you are acutely aware of your speed. The reason for this is because the motorcycle is very badly designed, aerodynamically speaking. Unlike the car or the plane which are designed to be streamlined, the motorcycle disrupts the air through which it travels and the rider feels a force due to the collisions of the air molecules on her body. Cars and planes are mercifully smooth and enclosed, and such forces are therefore not felt.

[3] This is a result of a very deep and fundamental concept that has nothing to do with Biology, but which is an essential part of Physics which we will study next week.

Question: Show that the dimensions of force are given by

$$[F] = MLT^{-2} \tag{9}$$

Force is thus measured in **units** of 1 kg m/s^2, otherwise called 1 newton (after the big man himself).

The most instructive way of writing Newton's Second Law is

$$a = \frac{F}{m} \tag{10}$$

as it shows that the acceleration a body experiences is related to the force (an external agent of change) and the body's own mass.

2.1 Mass

Imagine you are in free space, and you had two objects (as shown in Figure (2)) that you are trying to push. The first object has a mass of (say) 50 kg, and the second has a mass of 100 kg. You apply on each of them a constant force of 120 newtons, that is 20 kg m/s^2.

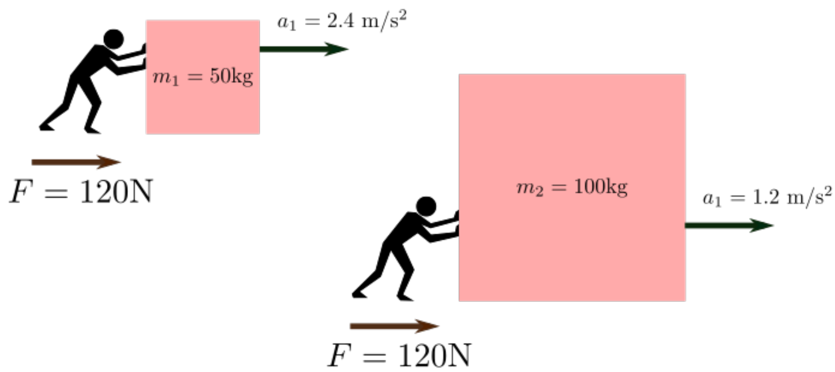


Figure 2: The same constant force acting on two objects, one twice as massive as the other, will produce two times greater an acceleration on the lighter object than on the heavier one.

Let us calculate the accelerations of each of these objects: for object 1, we apply Newton's Second Law to get that the acceleration is[4]

$$a_1 = \frac{F}{m_1} = \frac{120 \text{ kg m/s}^2}{50 \text{ kg}} = 2.4 \text{ m/s}^2$$

[4] I have purposely chosen numbers that don't divide exactly, since I want those of you uncomfortable with mathematics to practice a little bit.

Applying this law to the second object, we get

$$a_2 = \frac{F}{m_2} = \frac{120 \text{ kg m/s}^2}{100 \text{ kg}} = 1.2 \text{ m/s}^2$$

Thus, the accelerations are not the same. More massive objects accelerate less than less massive ones, when acted on by a constant external force.

2.2 Circular Motion

If we place ourselves in an inertial frame, and we see an object accelerating, then it must also be feeling a force. However, we must realise that acceleration deals with *velocities* not speeds. Consider the example of an object rotating in a circle. At some arbitrary instant of time, the object is moving at some velocity (say v_0). The force is always at 90° to the velocity (as you can see from the Figure (**??**)) and it can be shown mathematically that this means that it is only the *direction* of the velocity that changes at every instant, not its magnitude. We can use this to calculate the force that is experienced by such an object. The actual derivation requires either a lot of geometry or some calculus, but let us perform an "order-of-magnitude" estimate.

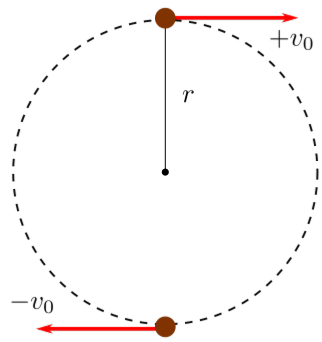


Figure 3: An object moving in circular motion: it's velocity shifts from $+v_0$ (i.e., speed of v_0 to the *right*, to $-v_0$ (i.e. speed of v_0 to the *left* when it moves from the top to the bottom of the circle.

We know that acceleration is given by

$$a = \text{ acceleration } = \frac{\text{change in velocity}}{\text{change in time}} = \frac{\Delta v}{\Delta t}$$

In the two points shown in the diagram, the velocity changes from being $+v_0$ (i.e. a speed v_0 going towards the right) to $-v_0$ (i.e. a speed v_0 going towards the *left*). The total change in velocity is thus $(+v_0 - (-v_0)) = 2v_0$.[5]

[5]If you are confused about this, imagine that someone moving to the right at 20 km/hr suddenly starts moving to the left at 20 km/hr. What is the total change in his velocity?

What is the total time taken to go from the top to be bottom? A naive way to solve this (there might be others) would be to take the total distance the object is covering (half the circumference of the circle with radius $r = 2\pi r/2$) and divide it by the "speed" v_0. Thus,

$$\begin{aligned} \Delta v &= 2v_0 \\ \Delta t &\sim \frac{\pi r}{v_0} \\ \implies a &\sim \frac{2}{\pi}\frac{v_0^2}{r} \end{aligned} \tag{11}$$

The actual answer is not too far off: for circular motion, the acceleration is

$$a = \frac{v_0^2}{r}$$

Thus, from Newton's Second Law, in order to keep an object of mass m in uniform circular motion at some speed v, one would need to exert a force from the centre which is equal to

$$F = ma = \frac{mv^2}{r}$$

2.3 Elevators

Another example of when this happens is what happens in a lift. Imagine you are standing in a lift that's initially at rest,[6] and you feel a force acting on you from the ground. It's important that you realise this: the force that you feel is acting *upwards* against your feet. Let us first understand why you feel this: the force of gravity is indeed pulling you down, and given half a chance, it will make you accelerate towards the centre of the Earth. This is what happens when you step off a high table, for example – the force of gravity accelerates you downwards. However, as you stand still on the elevator, you are certainly not accelerating. The reason for this is because the ground is producing an opposing force which *counters* gravity. As a result, both these forces cancel out and you stay at rest. Note that despite these two forces being equal and opposite, they are *not* a result of Newton's third law, since both the forces act on the same object (in this case, you).

Thus, you feel the floor exerting a force on you which is equal to your weight mg, upwards. Now imagine that the elevator starts to accelerate upwards with some acceleration a: in addition to the force that the floor is exerting on you to counter gravity, there is now *another* force that you feel due to the acceleration of the lift. This force that you feel is equal to ma (as your mass is m and you are being accelerated), and acts in the same direction as reaction force to gravity. Thus, the net force that you feel is

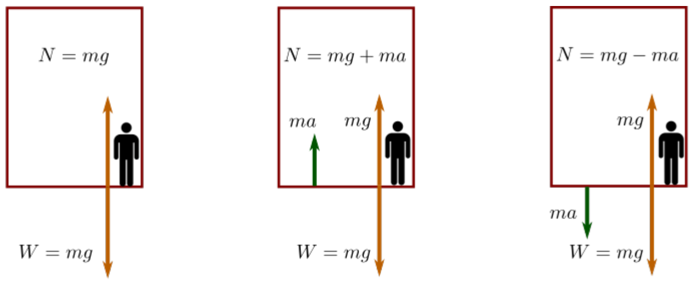
$$F_{\text{net}} = mg + ma \text{ (both forces act in the same direction, and hence add)}$$

As a result, you feel a little heavier as the lift takes off, since the floor pushes up on you a little more than usual; you can actually test this using a weighing balance, or by getting an "accelerometer" app for your phone.

[6] I'd advise you all to go out and actually perform this experiment.

The lift then moves with a constant velocity, so you no longer feel heavier (since now the floor's force is only countering gravity), until it starts to decelerate when you reach your destination. As the lift decelerates, the lift's floor is being "pulled down" with an acceleration a, which reduces the force your feet feel on the floor. Thus, the net force that you feel in this case is

$$F_{\text{net}} = mg - ma \text{ (both forces act in different directions, and hence subtract)}$$



(a) Lift standing still (b) Lift accelerating up (or decelerating down) (b) Lift decelerating up (or accelerating down)

Figure 4: In each of the cases, the weight force W due to gravity always acts downwards. What changes is the force of the lift's floor on us N. When lift stands still, this is just mg, since we are not accelerating. As the lift accelerates upwards, the floor provides an additional force ma which we feel. As it decelerates to a stop, a force ma is felt in the opposite direction.

Similar processes happen when you leave a floor and travel downwards in a lift. The lifts acceleration acting downwards *reduces* the force the floor acts on you, and you feel slightly lighter. As you reach the ground, the lift decelerates, providing an acceleration *against* your velocity, this time making you feel a slightly greater force from the floor. If the lift's cable were cut, both you and the floor would accelerate at the same rate, and you would feel no force from the floor. This experience of "weightlessness" might be exhilarating if it weren't for the anticipation of the swift end awaiting you at the bottom!

3 Gravity

The above statement might have seemed a little strange: why would the elevator and you both accelerate downwards at the same rate? It seemed, from our earlier analysis, that the acceleration of a body under a constant force should be more for a lighter object than a heavier one. This is true for *constant* forces, but not for gravity, as we shall explain.

Let us re-examine the equation of the Second Law:

$$a = \frac{F}{m}$$

As we are now dealing with gravity, let us call this acceleration by its own special name "g", and the force F_g (g for gravity). Thus,

$$g = \frac{F_g}{m}$$

From Galileo's experiments on inclined planes, we know that the **acceleration due to gravity is independent of the mass of the object that is falling**. But from the above equation, it should be clear that if the force of gravity were constant on all masses, this would not be true.

Thus, the force of gravity must **depend on the mass** of the object. In other words,

$$F_g \propto m$$
$$i.e.\ F_g = \text{constant} \times m$$

If this were the case, then if we replaced F_g in the above equation,

$$g = \frac{\text{constant} \times m}{m} = \text{constant}$$

Let this sink in: a more massive object experiences a greater force than a less massive one, but the acceleration that they both feel is exactly the same. This should be clear, if we are to proceed, so take some time. Ready? Good.

Now, we ask ourselves *why* there is such a force. Newton's genius was to realise that this force was due to the mass of the Earth, M_E. We now remember Newton's Third Law: forces come in pairs. If the Earth is attracting the object (say, an apple) with a force F_g, the Third Law says that the apple must be attracting the Earth with the same force (but in the opposite direction). Thus, the Earth experiences a force $-F_g$, and "falls" towards the apple.

This force cannot be independent of the Earth's mass, as there is nothing differentiating the Earth and the apple other than the numerical amount of mass they possess. We could think of a thought-experiment, keeping the apple as it is and "raising" the Earth a certain height above the apple and letting go. This should be exactly the same situation as given above (since there too the Earth will experience a force of $-F_g$ from the apple, and the apple a force of F_g from the Earth).

Thus, it must be that

$$F_g = \text{another constant} \times M_E$$

Question: The image of the Earth falling towards the apple is understandably unbelievable. Let's try and put some numbers to it: say you are a 100 kg, and falling towards the Earth. It is exerting a force of 100 kg $\times$ 10 m/s^2[a] on you. By the Third Law, the Earth must also be experiencing a force due to *you* of 1000 kg m/s^2. What is the acceleration of the Earth?
Hint: The mass of the Earth is 6×10^{24}kg.

If you're enthusiastic: Show that in a couple of seconds, the Earth "falls" towards the apple by a distance much, much, smaller than an atom.

[a] I have used g=10 m/s^2, you may use 9.8 m/s^2 if you wish.

Let's have a look at this again:

$$F_g \propto m$$
$$F_g \propto M_E$$

In other words, if we kept the mass of the Earth constant, and doubled the mass of the apple, the force would double. Going the other way, if we kept the mass of the apple constant, and doubled the mass of the Earth, the force would *still* double.

Question: Convince yourself that this means:

$$F \propto m \times M_E \tag{12}$$

Newton's generalisation of this law, possibly the greatest generalisation achieved by the human mind, was to imagine that it was this very same law that governed the motions of planets. From the studies of Kepler and Brahe, he knew that the force on different objects seemed to *reduce* as the distance increased, so he could say that

$$F \propto \frac{mM_E}{r^n} \tag{13}$$

where r is the distance between the two objects (in this case the apple and the Earth) and n is some number – the larger n is, the faster the effect of the force "dies out" over long distances. This is as far as we can go using only logic. Newton then postulated a constant (G) that multiplies the right-hand side of the above equation which would be the same for *any* two masses (m_1 and m_2) that attract each other through gravity:

$$F = G \times \frac{m_1 \times m_2}{r^n} \tag{14}$$

Question: Convince yourself that if you have two masses m_1 and m_2 attracting each other by gravity, that this means that the **acceleration** (not Force!) of m_1 depends on the mass of m_2, and the acceleration of m_2 depends on the mass of m_1.

We can now get the number n by using the fact that the apple falls towards the Earth due to the same force that keeps the moon in orbit around the Earth.

Question: Show that the acceleration of the apple due to the force of the Earth is given by

$$a_{\text{apple-Earth}} = G\frac{M_{\text{E}}}{R^n}; \quad R = \text{Radius of the Earth}$$

and that the acceleration of the moon due to the force of the Earth is given by

$$a_{\text{moon-Earth}} = G\frac{M_{\text{E}}}{(r_{\text{moon-Earth}})^n}$$

Question: Show that this means that

$$\frac{a_{\text{moon-Earth}}}{a_{\text{apple-Earth}}} = \left(\frac{R}{r_{\text{moon-Earth}}}\right)^n$$

Question: Taking the acceleration of the moon to be 0.00272 m/s^2, and the acceleration of the apple to be 10 m/s^2, and the distance of the moon from the Earth to be $60R$, show that $n = 2$. The force of gravity is thus known to follow an inverse-square law, as it falls off as the inverse square of the distance between the objects.

On the surface of the Earth, the distances between objects and the centre of the Earth are more or less constant, and so we can see that all objects on the surface of the Earth experience a force (due to the Earth) of

$$\begin{aligned} F_g &= m_{\text{obj}} \times \underbrace{\left(\frac{GM_E}{R^2}\right)}_{g} \\ F_g &= m_{\text{obj}} \times g \end{aligned} \tag{15}$$

Question: What are the dimensions of G?

Question: Suppose the acceleration due to gravity on a planet g_p depended on the planet's density ρ, its radius R, and the gravitational constant G, find how g_p depends on these parameters. i.e. assume

$$g_p \sim G^a \rho^b R^c \tag{16}$$

and find a, b, and c.

4 The Conservation of Energy

We have seen that on the surface of the Earth, the force of gravity is nearly constant, and equal to mg. When we move objects up and down, we are doing some work against this force.

It may not be obviously evident to you that the amount of work you do to lift an object depends only on the *vertical* distance it is displaced. Work is only done when an object is displaced against a force.

You can see this by imagining a perfectly frictionless table on the surface of the Earth: moving an object on such a table will require less and less work, as the force of friction is reduced (indeed, it would be a very troublesome thing, as in the ideal case, even a slight nudge would send the object sliding away!).

Another way you can see this is that if you have the misfortune to live on the 7th floor, it is quite difficult to get up there, however once you *are* there, it's not harder to walk around, or lift objects. In fact, without looking out a window, there's no way you'd be able to tell if you were on the ground floor, the third, or the seventh!

When you apply a force over a given distance, you do **work**. By lifting an object to a height h above the ground, I have applied a force equal to its weight (mg) over a distance h. The work I have done is given by:

$$\begin{aligned}\text{Work} &= \text{Force} \times \text{Displacement} \\ \text{Work} &= mgh\end{aligned}$$

But when I have raised the object, where has this work gone? It has gone into a state of limbo, a state of "potential" work. It certainly still has the potential to do work, as I could drop the object and it would fall back on my foot causing me no small distress. It does this by changing its configuration. In other words, the work that I have done has gone into changing the **configuration** of the system. By doing work, we change a quantity of the system known as its **energy**. But lifting an object a certain height, it is said to have **potential energy**.

$$\text{Potential Energy due to gravity} = mgh \tag{17}$$

Of course, we could also have an object on the surface of a frictionless table on the Earth sliding with a uniform velocity v. As it is not changing its height, clearly its potential energy (as defined above) is the same. However, in trying to stop it, we must change its velocity (and therefore impart and acceleration the object). This requires the use of force, over a certain distance until it comes to rest. Thus, the object *must* possess some energy by virtue of its motion. We call this **kinetic energy**. Mathematically, it can be shown that:

$$\text{Kinetic Energy of an object moving at velocity } v = \frac{1}{2}mv^2 \tag{18}$$

This also explains why when we pick an object up from rest and raise it up to a height to let it go, it speeds up as it falls. This process is a conversion of energy from potential to kinetic. As the object falls, it reduces its potential energy, and thus has to increase its kinetic energy since the total energy in a system is always conserved.

Question: Using the conservation of energy, show that the maximum height a ball thrown with a velocity u can reach is:

$$h_{\max} = \frac{u^2}{2g}$$

Such conservation laws are very useful in Physics, as they allow us different ways of solving a problem, as we shall illustrate while trying to respond to Aristotle's old observation: "Heavier objects fall faster than lighter ones".

5 Drag

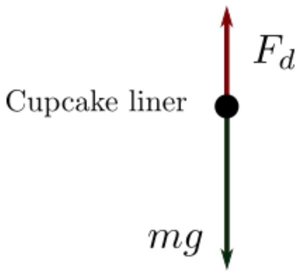


Figure 5: A force-diagram: gravity acts downwards, and the drag force acts against its motion.

Imagine a cupcake liner, as shown in Figures (5) and (6), released from a certain height. As the liner falls, it experiences two forces: one from gravity, and the other from friction with the air. This second force is what we call a "drag force", which we will denote by F_d.

5.1 Calculating F_d

Calculating the drag force might seem like a formidable task, but it can be greatly simplified using some assumptions.

1. First, we imagine the liner at some point in its trajectory, moving at some velocity v.
2. We assume the air underneath the liner to initially be at rest before it comes in contact with the linear, and then to be dragged along with the liner at the same velocity v as the liner after contact. (It may be clearer for you to imagine the particles of air to be little beads that stick to the liner as it passes).
3. We imagine a tiny instant of time Δt which is so small that the acceleration of the liner due to gravity can be ignored, and only its velocity contributes to changing its position.

Once you have convinced yourself of these assumptions, we are ready to start. We begin by realising that by bringing the air molecules up to its own speed, the liner has given them momentum (and therefore lost momentum itself, since momentum must be conserved). Let us thus start off by calculating the amount of momentum given to the air.

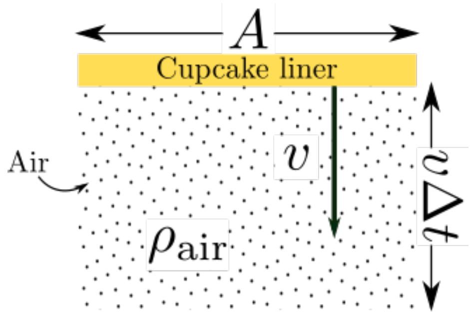


Figure 6: The cupcake liner falling through air in an instant Δt covers a distance $v\Delta t$.

$$\Delta p_{\text{air}} = \text{Momentum received by air} = M_{\text{air}} \times v - M_{\text{air}} \times 0^{7}$$

But what is M_{air}? We can think of the liner as sweeping out some volume V in time Δt, and multiply this by the density of air ρ_{air} to get the mass of air. Convince yourself that if the area of the bottom of the liner is A, the volume of air swept out by the liner is

$$\text{Volume of air} = A \times (v\Delta t)$$

Thus, from Equation (5.1) the total momentum given to the air in a time interval Δt is

$$\Delta p_{\text{air}} = (\rho_{\text{air}} \times Av\Delta t) \times v = \rho A v^2 \Delta t \tag{19}$$

By the conservation of momentum, it must be that the total momentum *lost* by the liner Δp_{liner} is equal and opposite.

$$\Delta p_{\text{liner}} = -\rho_{\text{air}} A v^2 \Delta t \tag{20}$$

Thus, this "drag" force on the liner, defined as the rate of change of momentum due to collisions with air, is given by

$$F_d = -\frac{\Delta p_{\text{liner}}}{\Delta t} = -\rho_{\text{air}} A v^2 \tag{21}$$

We now have enough information to attempt a solution.

5.2 Solving the problem using forces

We can now ask ourselves the same question as before: what is the net force on the liner?

$$F_{\text{net}} = mg - \rho_{\text{air}} A v^2 \tag{22}$$

[7] As the air is initially at rest, its initial velocity is 0.

at every instant of time. However, as the object falls, its velocity v increases. Thus, there is some velocity $v = v_t$ when the right hand side of the equation is zero! This may not be evident to you, take some time to understand it.

At this velocity v_t, the net force on the liner is zero, and so its acceleration is zero, and hence **it stays at the same velocity** v_t! We can now easily calculate v_t, as by definition, at $v = v_t, a = 0$.

$$\begin{aligned} a &= 0 \text{ (when } v = v_t) \\ &= mg - \rho_{\text{air}} A v_t^2 = 0 \\ \implies v_t &= \sqrt{\frac{mg}{\rho_{\text{air}} A}} \end{aligned} \tag{23}$$

Thus, v_t – the *terminal velocity* that the object attains – *is* dependent on the mass of the object. Heavier objects have a higher terminal velocity and thus attain it much later than lighter objects. A feather's terminal velocity is so small that the instant it is released, it attains it, and from then on it floats down at this constant velocity. However, a book's terminal velocity is so much larger that you'd have to drop it from a rather tall building for it to reach this velocity before hitting the ground!

We can also see that this velocity depends inversely on the medium in which the object is dropped. By dropping it in a very dense medium, the terminal velocity is reduced, and therefore attained much sooner. This explains why objects dropped in water fall slower than those dropped in air.

5.3 Solving the problem using energies – optional

We could now solve the same problem using the energy viewpoint. When we drop the object from some height h, it has some potential energy. As it falls, we've seen that its potential energy converts into kinetic energy. However, we know that after a point its kinetic energy stays constant (since it does not change its velocity!). However, its potential energy keeps decreasing (as it's falling). We seem to have a bit of a problem: energy cannot disappear, yet here it seems that as an object falls with a constant speed, its potential energy reduces, but its kinetic energy stays the same.

The solution is – of course – that there is the drag force F_d against which such motion is occurring. Moving an object against any force requires work, and thus the expenditure of energy.

Let us assume that the object has reached a velocity v_t, such that it is no longer increasing its velocity. In this case, all the potential energy the object loses must be used to overcome the frictional drag! If the object falls a certain distance (say y), then it has lost a potential energy

$$\text{Loss in potential energy} = mgy$$

However, at the same time, it has moved a distance y against the drag force F_d. Thus, the energy to move it against the drag force is

$$\text{Energy required to move it against drag} = F_d \times y = \rho A v_t^2 y$$

Equating these two energies, we see that y cancels out on either side, and that

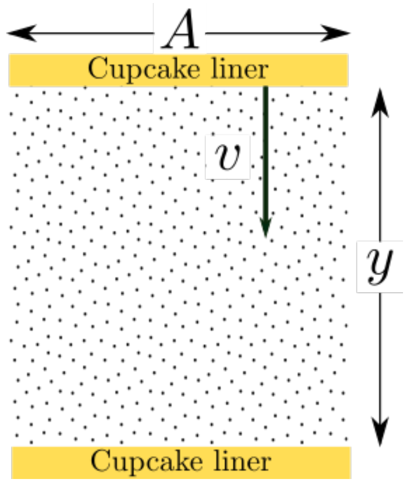


Figure 7: As the liner falls a distance y, its potential energy drops by the same amount as the work done against the drag force F_d.

$$v_t = \sqrt{\frac{mg}{\rho_{\text{air}} A}} \tag{24}$$

just as before.