Lecture 7 – Waves

Physics for Pedestrians

6th August, 2019

1 Oscillatory Behaviour

When any stable mechanical system is disturbed, Nature's response is *simple harmonic motion*: the point particle oscillates about an equilibrium point, very much like a mass on a spring oscillates about its equilibrium height. For a single oscillator, that's as far as it goes, but now consider a bunch of linked oscillators: a disturbance in one is passed on to the next, and this continues down the line. This is the essence of a "mechanical" wave. Sometimes, the individual oscillators are easy to discern, as in the case of coupled pendula, while in other cases (such as in water) the waves themselves are easy to see but their individual components are indiscernible. Waves can thus be thought of as a *disturbance* in a continuous medium.

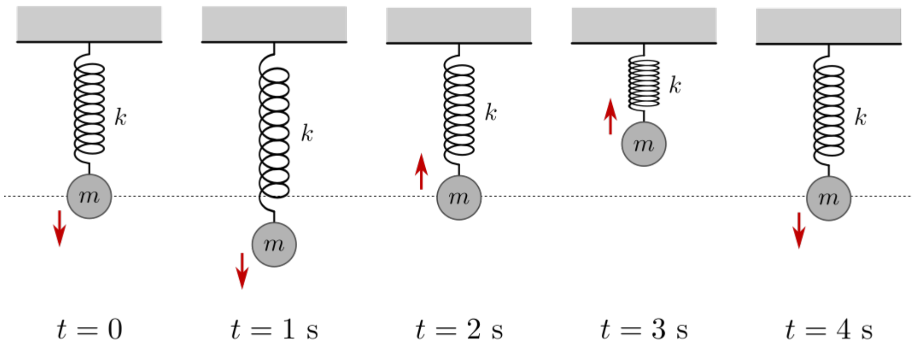

Figure 1: A single oscillator is hit at $t = 0$. As time goes by, it oscillates about its equilibrium point. After a time $t = 4$ s, it comes back to its initial state, only to repeat the process again. It thus has a *periodicity in time* of $T = 5$ s.

1.1 Waves

We are exposed to waves throughout the day, some of them much more exciting than others. Sound, in particular, is a special type of wave that is caused when the molecules of air compress and rarefy, propagating a disturbance in pressure. This disturbance is then picked up by your ears and interpreted by your brain as "sound".

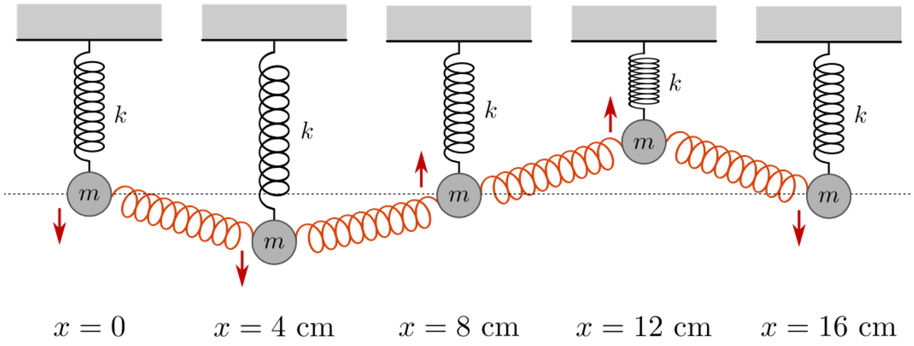

Figure 2: A snapshot taken at some instant of time of a set of oscillators arranged regularly in space and connected by springs. As you move along x, the oscillators are displaced differently around the equilibrium point. After a distance $x = 16$ cm, the pattern repeats. It thus has a *periodicity in space* of $\lambda = 16$ cm.

Waves have certain properties that make them very different from particles. The disturbance has a maximum displacement or intensity as the wave moves along, and a definite time for the completion of one cycle. A wave thus has a *periodicity in time* (the disturbance comes back to it's original state after some time) known as the time period T, and a *periodicity in space* (the disturbance repeats itself after some distance in space) known as the wavelength λ.[1] We can now use the simple definition of velocity to define the velocity of a wave as it propagates over one full cycle of space and time:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{change in time}} = \frac{\lambda}{T} = f \times \lambda$$

where we have defined a quantity known as the **frequency** of the wave as

$$f = \frac{1}{T}$$

The velocity of a wave in a medium is determined solely by the material composition of the medium (the same disturbance will cause waves that appear different in different media – air and metal being rather dramatic examples).

Question: What are the dimensions of f?

Question: As the velocity of the wave is determined by the medium, show that if a waves frequency increases, its wavelength must decrease. i.e. show that

$$f \propto \frac{1}{\lambda} \tag{1}$$

[1]Pronounced “lamb-da”.

1.2 Energies of Classical Waves

Such "classical" waves (which may be thought of as collections of oscillators) also possess an energy. The energy in such systems can be calculated and is found to be proportional to the amplitude. This should not be too surprising to you: it's much harder (i.e. you need a lot more energy) to make a skipping rope oscillate with a large amplitude rather than a very small amplitude. Doing the mathematics also proves this:

$$E \propto A^2$$

For the enthusiastic reader

Those of you familiar with simple harmonic motion should not be too surprised by this. The energy of a simple harmonic oscillator is given by

$$E = \frac{1}{2}kx^2$$

where x is the displacement. The equation for the wave is just a generalisation of this idea.

1.3 Diffraction and Interference

Waves exhibit some very interesting phenomena. Consider a wave that is incident on an aperture. The wave will be seen to bend around the aperture, as you saw in the demonstration in class. This principle is actually something you have seen or experienced often, though you may not have realised it. Consider someone playing music in their room, with the door closed: you might not be able to hear it while walking past the room. However, if that person where to open their door with the music playing, you could hear it not only when directly in front of the door opening, but also on a considerable distance down the hall to either side. This is a direct effect of diffraction. When sound passes through an opening, it bends.

Similarly, when two waves travel towards each other in opposite directions, they briefly cancel out: this is known as interference. The crest of one wave meeting the trough of the other and cancelling out is the epitome of wavey-ness: waves are disturbances and disturbances can cancel out, unlike particles. This has been represented in Figure (3).

Question: Can you use Figure (3) to explain why two cell-phone towers should not be placed too close to each other to ensure good reception in the neighborhood?

2 Light as a wave

The question then arose in the 17th century as to the nature of light. One theory, put forward by Descartes and championed by Newton, was that light is constituted of particles (or corpuscles, as he called them) which travel in a straight line with energy and momentum. The main reason for Newton's belief in this was because light was found to travel in a straight line, unlike most waves

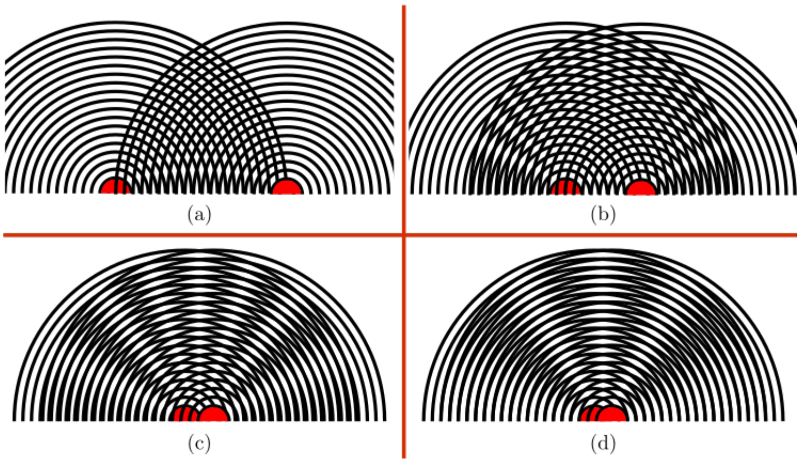

Figure 3: A spatial representation of two-slit interference: as the two point sources (depicted as red dots) approach each other, the waves they create (depicted by the semicircles) *interfere* with each other, producing bands of light and dark. The separation between these bands *increases* when the sources are close, and *decreases* when they are far apart. Such patterns are known as moiré patterns.

known at the time. In particular, it was known to bend into media, through a process known as refraction, while continuing to travel in a straight line before and after. Newton's success with Gravity had assured his reputation, and it did not seem wise to question him.

The other theory, suggested by Huygens, was that light was a wave. The difference between these two theories was clear: in Newton's theory, when light entered a denser medium from a rarer medium, the corpuscles would speed up – the speed of light should be greater in denser media. However, Huygens' wave theory predicted exactly the inverse: the speed of light should slow down in denser media. And this was indeed the case. However, while Huygens managed to prove this, it was still not a definitive proof as he hadn't *shown* that light was a wave, just that it moved in way that agreed with wave mechanics. The actual "proof" came in the way of another experiment.

2.1 Thomas Young's Double Slit Experiment

Young did a spectacular experiment that we still use in laboratories today. He realised that the real difference between "particles" and "waves" was the fact that waves exhibited both interference and diffraction. Since he believed that light was a wave, he reasoned that some sort of interaction would occur when two light waves meet. He performed a very difficult experiment with two small closely spaced slits through which he passed sunlight. If it truly acted as a particle, he would just see two bright bands. If, on the other hand, it behaved as a wave, it would produce a specific "interference" pattern which is very recognisable.

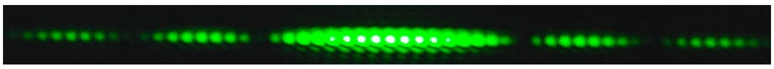

Figure 4: A double slit experiment conducted with a green laser. The pattern produced is very similar to that shown in Figure (3).

The result, of course, was an interference pattern which proved quite definitively that light was a wave, and this settled the argument for at least a hundred years, until the begining of the 1900s when the theory of Electromagnetism was established on sound theoretical and experimental evidence. It was found that light was an electromagnetic wave which travelled at a specific speed c. However, the theory had a peculiar but inescapable result:

> Accelerated charges emit electromagnetic radiation (light).

2.2 Blackbody Radiation

In the beginning, there was light. Eventually, there was the light bulb. Max Planck, a German physicist, was attempting to study how light was produced by such objects which glowed when heated.

The light bulb works as most light producing devices did at the time: through thermal radiation. Thermal radiation is the conversion of heat energy into electromagnetic energy. Ultimately, the object producing the light is composed of many different particles that are jiggling about at some speeds. How fast the particles jiggle about on an average is known as the "temperature" of the material. When we heat an object to higher and higher temperature, the average kinetic energy of its constituents due to this "jiggling" increases, leading to stronger and more frequent collisions between them.

During a collision, atoms and molecules which have charged particles within them abruptly change their velocities, which means they **accelerate**.

> The collisions of the atoms or molecules within a material cause the charges to accelerate and hence emit radiation in the form of light. Such radiation is known as "Blackbody radiation".

This radiation is found to **only** depend on the temperature of the emitting object, and it has a very distinctive curve, shown in Figure (5).

The question that Planck was trying to answer was roughly this: why is it that objects that are heated glow more red than blue? If you look at the spectra for most materials, you'd see that there is usually very little of the colours[2] blue and below in wavelength. Classical theories could not explain this satisfactorily.

Planck's intuition told him that if the energy of light depended on its **frequency**, then high frequency (or low wavelength) light would not be emitted as much at lower temperatures (as it would cost more energy to emit them), and then hotter objects would glow "bluer". Thus, he postulated that

$$\begin{aligned} E &\propto f \\ E &= hf \end{aligned} \tag{2}$$

where h is a new constant which relates the two quantities, now known as **Planck's constant**.

[2] I use the word colours loosely: fundamentally, we are talking about electromagnetic waves that exist whether we perceive them with our eyes or not!

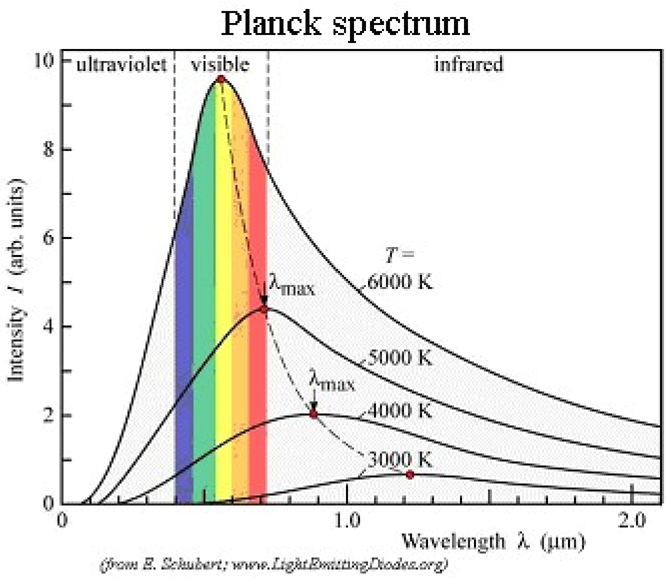

Figure 5: The Planck Spectrum for different temperatures. The form of the curve is specified completely by the temperature.

Question: Show that the dimensions of Planck's constant are

$$[h] = ML^2T^{-1}$$

Question: Show that h has the same dimensions as another physical quantity known as angular momentum L, where

$$L = p \times r = mvr$$

This worked fantastically, and the spectrum was perfectly described by Planck's hypothesis. Thus, with the introduction of this new constant h (which was a new fundamental constant of Nature) a new field of physics was born, that of Quantum Mechanics. The idea was simple: energy comes in discrete packets. But the consequences were profound.

3 The Photoelectric Effect

3.1 The Atom

This had consequences in other area as well. By this time it was reasonably well established that atoms existed. J. J. Thomson was performing experiments on what were known as *cathode rays*: negatively charged particles that were emitted from cathode tubes, which caused electricity. Thomson realised that these "electrons" were present within atoms, and (since atoms were clearly overall

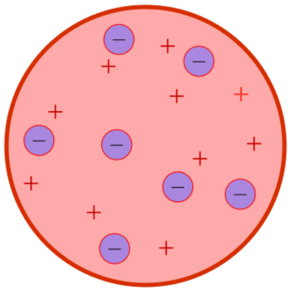

Figure 6: Thomson's plum-pudding model of the atom.

neutral) proposed a "plum-pudding" model of the atom, with a positively charged pudding and electrons as negatively charged plums.

His student, Ernest Rutherford, tried to test this theory. He took a very thin sheet of gold foil and bombarded it with heavy, positively charged α[3] particles. As expected, most of them cut through the atoms like pudding. However, to his surprise, some of the particles rebounded! This was a huge shock for Rutherford. As he said himself, *"It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."*

Rutherford understood that this meant that atoms were mostly empty space. Since some of the α particles rebounded, it must mean that the positive charge of the atoms is concentrated in certain locations, so that it was sufficient to repel the highly charged α particle. Furthermore, since he did not observe a huge recoil of the atoms, it must mean that most of the mass of the atom was also concentrated at this central location. This led to the model of the atom you have seen in school, with the heavy nucleus at the centre and the electrons orbiting it at different locations: the "solar system" model of the atom, if you were. There seemed a pleasing symmetry between the large-scale and small-scale physics. But of course, there was a problem with this, which we'll talk about later.

3.2 Of light and matter

Frank Hertz, another German physicist, was performing experiments with the cathode tubes, and he found something very strange. Light, when shone on these cathods, was absorbed and the cathodes emitted electrons! What was even stranger was that it seemed that it was not the *intensity* of the light that affected the eenrgy of these electrons. Of course, light with a lower intensity did produce *fewer* electrons, but the *energies* with which these electrons left the cathode was the same. What was even stranger was that it was the *colour* of light that seemed to affect the energy! At the same intensity, blue light seemed to emit electrons with a higher energy than red light!

So, to summarise, this "photo-electric" effect had two strange results:

1. The intensity of light (which depends on its amplitude squared) only seemed to affect the **number** of electrons emitted, not their energies.

[3] "Alpha"

2. The colour of light (which depends on its wavelength or frequency) seemed to affect the **energy** of the electrons emitted, not their number.

This was contrary to classical waves, as in classical waves (as we have already seen) the energy is proportional to the intensity (the square of the amplitude). However, it was Einstein's genius to combine this experimental result with Planck's earlier hypothesis to suppose that maybe the energy of light depended on the **frequency**.

Suppose the electron is trapped in the atom, and needs energy equal to ϕ (known as the "work-function" of a material and different for different materials) to escape. Say the electron absorbs ultraviolet light arriving in a bundle of energy $E = hf$. Then, it will have enough energy to escape the atom, and all the remaining energy will go into its kinetic energy. Thus,

$$K = hf - \phi \tag{3}$$

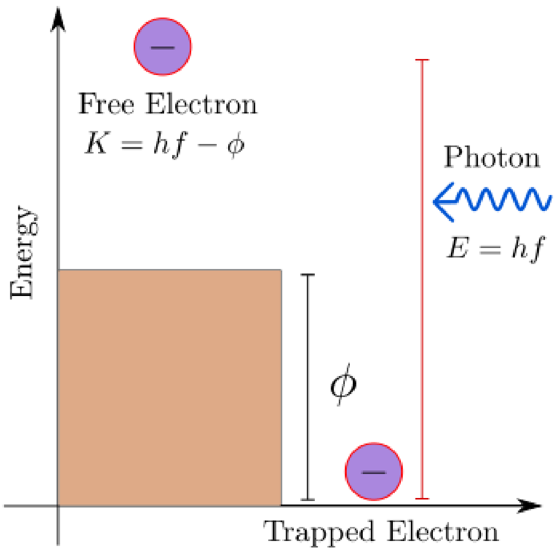

Figure 7: An electron is trapped in at atom by an amount of energy ϕ. If it absorbs a photon of energy $hf > \phi$, it can escape the atom, and all the remaining energy will go into its kinetic energy K.

By measuring K and f and plotting a graph, we can find h and ϕ. This was done by Robert Millikan, and it gave a reliable method to measure Planck's constant (h).

The consequence was inescapable: light comes in lumps – particles called photons.