Lecture 9 – Astronomy and Cosmology
How Gravity Acts on Different Scales

Physics for Pedestrians

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The force that organises the universe is gravity. There are four fundamental forces in nature: of these the strong and weak forces act only over short distances; the electromagnetic force can act over great distances, but because there exist positive and negative charges, whose influences tend to cancel each other out, it is effectively insignificant over long distances. This leaves only the gravitational force. Everything in the universe exerts gravitational force and is susceptible to it. As a result, the only force that matters on the scale of the universe is gravity.

1 The Densities of Solids

At very short distances and masses, the force of gravity is virtually non-existent. The force that organises things at this scale is the electrostatic force. A curious consequence of this is that the densities of most solids are – up to a factor of 10 – the same.

Most solids have densities from 1 to 10 g/cc.

This can be understood in the following way: imagine that solids are composed completely of atoms that stick to each other. The atoms cannot be compressed any further, as their electron clouds repel other atoms' electron clouds. Thus, the density of the whole solid should be very close to the density of an atom!

The density of an atom is a curious thing: as Rutherford discovered, almost all the mass of the atom is concentrated at its centre, but it is the electron cloud's radius that gives it is volume. Thus, we now have the tools to estimate the density of an atom!

Question: Consider a simple hydrogen atom with one proton and one electron (orbiting at the Bohr radius r_1 defined in the previous lecture). Find the mass of the proton, call this m_p. Convince yourself that the volume of the atom is just $(2r_1)^3$ (you're assuming it to be a little cube). Show that the density of such an atom is roughly

$$\rho = 1.6 \times 10^3 \text{ kg/m}^3$$

2 The Heights of Mountains and the Shapes of Asteroids and Planets

Most of the major objects in the solar system – the sun, the planets, and the satellites – are spherical. There is, however, a group of objects that are not – the asteroids. Why are planets spherical whereas the asteroids are not? This has to do with their mass and size.

Any fluid agglomeration of matter tends to be spherical, because that is its lowest-energy configuration. This is why bubbles and drops of water are spherical. Solid matter that we encounter in everyday life remains non-spherical because of strong inter-molecular bonds that cannot easily be deformed. When the deforming forces are strong enough, however, these bonds can break, making solids fluid-like.

One consequence of this is that a mountain on a planet has a maximum possible height. If it exceeds this height, then its weight causes the molecular bonds at the bottom to deform and break, making the mountain sink into the planet. On Earth, this maximum height is about 10 km. On smaller planets this maximum height is greater and on larger planets it is smaller. Thus the larger an object, the more perfectly spherical it tends to be. On the other hand, if the the object is small enough, the "mountains" may become comparable in size to the object itself – then the object is effectively non-spherical; this is what happens in asteroids.

3 The Sun and Other Stars

The star is in some sense the fundamental constituent of the universe. Stars are formed in clouds of molecular hydrogen. When a portion of a cloud reaches a certain density its own gravitational force causes it to collapse. As it collapses, its temperature increases. Ultimately, the temperature at the centre of the collapsing object becomes high enough for hydrogen fusion to occur. This in turn causes an outward pressure that, when it is balanced by gravity, produces a stable object called a star. Because the amount of fuel available for fusion is finite, every star has a finite lifetime, during which it remains more or less stable. How long a star lasts depends on its mass. The larger the star the shorter its lifetime! The largest stars, which have masses a few tens of times that of the Sun, have lifetimes of the order of a few tens of millions of years. The Sun, on the other hand, as a medium-mass star, has a lifetime of about 15 billion years; one-third of its life is over. And the smallest stars, with masses about a tenth that of the Sun, have lifetimes of hundreds of billions of years.

The end of the life of a star can be quiet or very dramatic. The smallest stars just quietly burn out. A medium-mass star expands into *red giant* towards the end of their life; when our Sun becomes a red giant its outer surface will swallow Mercury. Then the outer part is sloughed off as a *planetary nebula* while the core becomes a *white dwarf*. A large star undergoes a dramatic implosion followed by an explosion called a *supernova*, which is one of the most dramatic events in the universe; a single supernova blast can produce more light than hundred or billions of stars.

Because of the very different lifetimes of small and large stars, a molecular cloud, in which stars of all masses are formed, is simultaneously a nursery and a cemetery: while the smaller stars in it are still in the earliest stages of formation the larger stars are already dying.

4 The Planets Around the Sun

When a portion of a molecular cloud collapses to form a star, in the initial stages the spherical core is accompanied by an equatorial disk of material (rather like the rings of Saturn). This happens because a cloud of gas typically rotates and thus has angular momentum. As it collapses, to conserve angular momentum it rotates faster, leading to the formation of a disk. Different portions of the disk then fragment and coalesce to form planets. This picture of planet formation explains why the planets all revolve in the same direction around the Sun and more or less in a plane.

As is well known, each planet revolves around the Sun in an elliptical orbit, with the the Sun at one focus. In fact the ellipticity of the orbit is very small for every planet other than Mercury – the orbits are effectively circular. The orbital velocity v that an object of mass m, e.g. a planet, must have to maintain a circular orbit at a distance of r from the Sun is given by the equation that balances centrifugal and gravitational tendencies (on the left and the right in the equation below):

$$\text{Centrifugal force} \longrightarrow \frac{m\,v^2}{r} = \frac{G\,M\,m}{r^2} \longrightarrow \text{Gravitational force,} \tag{1}$$

where M is the mass of the Sun and G is the universal constant of gravity.

Question: Show that this leads to the following relationship between orbital velocity and radius:

$$v = \sqrt{\frac{G\,M}{r}}. \tag{2}$$

This $v-r$ pattern is called a *rotation curve*. The rotation curve in which v falls as $1/\sqrt{r}$ corresponds to a centrally concentrated mass distribution, here the Sun.

(In the theory of orbital motion we have described so far we have pretended that the Sun is stationary and unaffected by the motion of the planets. Of course that cannot be true. If you think of a star and a single planet as a closed system, it is obvious that, because of conservation of momentum, the motion the planet must cause some compensatory motion in the star. It is very small, much smaller than the size of the star, but with modern technology one can detect this motion. It is this idea that has led to the discovery of *extra-solar planets*.)

The orbit of each planet would be perfectly elliptical and closed only if it is the only one orbiting the Sun. But each planet in the solar system experiences the gravitational force of the other planets as well. As a result, the orbits are not perfectly elliptical, and instead of being closed they precess. (These effects were too small to have been detected in the age of Tycho Brahe and Johannes Kepler.)

We say that the orbits undergo perturbations due to the other planets. These perturbations can be calculated using Newton's laws of motion and his law of gravity. By the the middle of the 19th century the theory of perturbations of planetary orbits was so well understood that any deviation from any of its predictions warranted a search for a cause.

At that time the perturbations of the orbit of Uranus were not completely explained by planets then known to exist. By studying these perturbations two astronomers, Urbain Le Verrier and John Couch Adams, predicted the existence of another planet in the neighbourhood. So precise were the calculations that the position of the unseen planet was predicted very precisely. Follow-up observations by Johann Gottfried Galle led to the detection of a new planet, later named Neptune. The prediction and discovery of Neptune was one of the great moments in the history of science, especially of Newton's theories of motion and gravity.

5 Einstein's Theory of Gravity

5.1 The Orbit of Mercury

One planet other than Uranus, it turns out, had perturbations in its orbit that could not be completely explained by the known planets. Urbain Le Verrier, after his success in predicting Neptune, was convinced that another unknown planet was responsible for the residual perturbations of Mercury. The truth was far more extraordinary. Albert Einstein presented in 1911 a new theory of gravity – called the General Theory of Relativity – in which it was not a force but the result of the curvature of space-time. Within the framework of this theory the orbits of the planets suffer an additional precession beyond that due to the other planets. The predicted precession due to Einstein's theory was exactly equal to the residual perturbation seen in Mercury.

5.2 Black Holes

Consider an object attempting to leave the Earth's gravitational pull. Such an object (say a rocket) would require to attain a velocity which was sufficient to wrest itself loose from this pull.

Question: Through dimensional analysis, argue that this velocity must depend on the Gravitational constant G, the mass of the Earth M, and the radius of the Earth R, and then derive the following equation (up to the constant which this method cannot tell you.)

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} \tag{3}$$

One of the strangest predictions of Einstein's theory of gravity is the black hole. A black hole is an object whose gravitational field is so strong that the escape velocity from it exceeds the speed of light. Black holes are expected to form when very large stars die. The gravitational waves that have been observed were from the coalescence of such stellar black holes.

Question: From the earlier question, ask yourself this: what should the radius of an object of mass M be, in order for its escape velocity to be c? Call this r_s, the Schwarzschild Radius.

Surprisingly, this is exactly the answer one gets if one solves Einstein's complicated Field Equations to find the same answer: if an object of some mass M is compressed within a sphere of radius r_s, it will *become* a black hole. (For the Earth, r_s is around 10mm, so fear not!)

Another region in which black holes are thought to exist are the centres of galaxies. The best evidence we have of this is the motion of stars near the centre of our own galaxy, the Milky Way. As explained earlier, the motion of planets around a star tells us about its mass. In the same way, the motion of stars around the centre of the Milky Way tells us about the mass of the object there. It is found to be a few million solar masses. Combining this with estimations of its size lead to the conclusion that there is a black hole at the centre of the Milky Way.

The black holes in the centres of many other galaxies are much larger. A large black hole at the centre of a galaxy leads to the formation of a disk of material around it as matter falls into it. There is tremendous friction in this *accretion disk*, and the electromagnetic emission from such an *active galactic nucleus* can be very powerful. As matter falls in through the equatorial disk, other matter is spurted out from the poles, forming huge jets which can be thousands of times the size of the galaxy itself.

All of this happens in the vicinity of a black hole but outside it. The inside and the outside of a black hole are marked not by a physical boundary but something called an *event horizon*; inside the warping of space-time is so severe that what happens in that region is beyond reach.

5.3 Gravitational Waves

If masses bend space-time then the motion of objects should cause the bending of space-time to propagate. And they do. The ripples in space-time are perceptible only for events like the coalescence of black holes. The recent discovery of gravitational waves by LIGO was one of the greatest achievements in the history of science.

5.4 Gravitational Lenses

In Einstein's theory gravity is caused and felt not just by massive objects but by every form of energy. So light, which is a form of energy, is bent as it moves past a massive object. This leads to astounding images of distant galaxies with foreground galaxies or galaxy clusters acting as *gravitational lenses.*

6 Elliptical and Spiral Galaxies

The two major types of galaxies in the universe are elliptical and spiral. Ellipticals are vast agglomerations of stars that do not show much overall rotation, whereas spiral galaxies have disk-like structures because of strong rotation. The Milky Way is a typical spiral galaxy. (The Sun goes once around the centre of the Galaxy every 220 million years.)

7 Flat Rotation Curves and Dark Matter

The rotation of stars and gas clouds around the centres of spiral galaxies makes it possible to plot their rotation curves. And, as with planets going around the Sun and stars going around the centre of the Milky Way, the rotation curve of a galaxy tells us about the distribution of mass in it. The big surprise is that the distribution of mass required to explain the rotation curve is very different from that corresponding to the visible stars. There appears to be an enormous amount of invisible matter needed in every galaxy to explain its rotation. It is called *dark matter*.

8 Clusters and Superclusters

Galaxies usually exist in *clusters*, which then form larger structures called *superclusters*. Dark matter is needed to explain their structure too. The average speed of random motions of galaxies within a cluster depends on the total mass in the cluster, and we find that the mass needed to explain the average speed observed is much larger than the mass one would associate with the light-emitting matter.

The distribution of galaxies across the universe – which we call large-scale structure – is such that a lot of dark matter is needed to explain it.

9 Distance Measurement

So far we have often spoken of measuring velocities but seldom of measuring distance. Velocity measurement along the line of sight is very easy and precise, because the motion of an object in this direction causes a shift in the pattern of light that it emits – what is called its spectrum – and this shift can be very precisely measured and mapped onto a recession velocity.

Distance measurement, on the other hand, is very difficult, because there is little about a heavenly object that reveals its distance from us. There are several methods by which distance is measured. The most important one is that which uses *standard candles*. A standard candle is a source of light whose intrinsic luminosity is known. Since brightness depends on intrinsic luminosity and distance, by bringing intrinsic luminosity and brightness together one can determine distance.

One of the most important standard candles for sources at medium distance is the *Cepheid variable star*. Such a star changes its luminosity in a periodic manner and we find that the period of variation depends on luminosity in a well-defined fashion. Thus, by observing the period of variation of such a star one can determine its luminosity, and thus its distance. A Cepheid variable in a nearby galaxy allows us to determine the distance to the galaxy.

10 Hubble's Law

Edwin Hubble was the first astronomer to measure the distances and recession velocities of a large number of galaxies at different distances from us. When he plotted velocity versus distance, he found something astonishing – the recession velocity of a galaxy increases in proportion to its distance. This implies that the universe is expanding – assuming that our position in the universe is like any other.

The theoretical framework for understanding an expanding universe was provided by Einstein's theory of gravity. The equations corresponding to an expanding universe had been worked out by a number of scientists including a French priest by the name of Georges Lemaître.

If the idea of an expanding universe is taken backward, it leads to a beginning in which the universe had infinite density. We now call this beginning the *Big Bang*.

At the beginning, in this understanding of the universe, all matter and other forms of energy were mixed up and indistinguishable. As the universe expanded, it cooled, and, at a certain point, stable hydrogen atom formed. From that moment onward, the matter and electromagnetic energy no longer "talked" to each other. The matter eventually formed the galaxies and other structures in the universe, whereas the energy just cooled. This energy forms a background that is to be found everywhere. It was detected in 1964 by Arno Penzias and Robert Wilson.

11 The Final Puzzle: An Accelerating Universe?

In the Big Bang theory of the universe, as it expands everything slows down. (It is similar to what happens when you throw something up – as it moves up, it slows down.) So there is no question of the universe accelerating. But, strangely, that is precisely what observations suggest is true.

A standard candle that can be seen over very large distances is a Supernova of type Ia. This kind of supernova forms when a white dwarf star in a binary system goes over the *Chandrasekhar mass limit* of 1.4 solar masses. It then becomes a supernova with a well-defined luminosity. By observing such supernovae at different distances, one can ask whether they are farther than they would be if the universe were slowing down. And we find that that they indeed are. This suggests that the universe is accelerating. No know form of matter can cause this phenomenon. So we call it *dark energy*. Most of the universe seems to be either dark energy or dark matter. Very little is actually visible.