Assignment 1: Dimensional Analysis and Differential Equations

Due: February 6, 2020 (Thursday)

Marks: 15

1 Dimensional Analysis

In the Discussion Session, we spoke in detail of Dimensional Analysis and how it can answer some apparently intractable problems quite simply. Explain your working out **very clearly**.[1]

(a) Imagine you spill a glass of some liquid on the ground. The height h of the puddle of liquid depends on the density of the liquid ρ, the surface tension s of the liquid, and the acceleration due to gravity g. Find a dimensionally correct formula for h using dimensional analysis. Taking the dimensionless constant to be of order 1, find h for water. **[2]**

(b) Consider a drop of liquid in free space (i.e., neglecting gravity). The drop takes on a spherical shape because for a given volume the sphere has the minimum surface area, and the spherical configuration minimises the energy associated with creating the surface. If this drop is disturbed, it starts oscillating. Find a dimensionally correct formula for the time period of oscillation in terms of the quantities that matter. **[3]**

2 Programming

(a) Start off with the simple harmonic oscillator that we spoke about in the Discussion Session. Now introduce a damping force that is of the form $F_d = -m\gamma\dot{x}$.

  (i) Write out the differential equation that this system satisfies. Identify the time scales in the problem. (You should be able to construct two quantities τ_1 and τ_2 of dimension time from the parameters of the problem.) **[2]**

  (ii) Write a code to solve this system using the leapfrog method, setting $\tau_1 = 1$ and $\tau_2 = 1$, with initial condition $x(0) = 1, v(0) = 0$. Plot the solution to this problem (i.e. plot $x(t)$) for a sufficiently large range of time. **[5]**

  (iii) Now choose a value of $\tau_1 \gg \tau_2$, and plot the solution to this problem for the same initial conditions. Now choose a value of $\tau_1 \ll \tau_2$, and do the same thing. **[2]**

  (iv) Comment on the any features in the above graphs that you find interesting. **[1]**

[1] You need to understand a simple concept that you may already be familiar with: the **surface tension** s of a liquid is the **energy** required to increase its **surface area** by one unit of area. This is all you need to know about it here.