Assignment 10: Partial Derivatives and Multiple Integrals

Suggested Submission: April 30, 2020 (Thursday) **Marks: 15**

1 Partial Derivatives and the Jacobian

(a) Consider the coordinates (u, v) in $\mathbb{R}^2$ that are related to (x, y) in the following way:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2uv \\ u^2 - v^2 \end{pmatrix}.$$

(i) Is the transformation linear? Why? **[1]**

(ii) Calculate the derivative (or the *Jacobian Matrix*) of this transformation. **[4]**

(b) Do some research on the **cylindrical** coordinate system, and show that the volume element is given by

$$\mathrm{d}V = \rho \mathrm{d}\rho \mathrm{d}\varphi \mathrm{d}z,$$

where the symbols represent their standard meanings. **[2]**

2 Multiple Integrals

(a) Evaluate the following integral: **[3]**

$$\int_0^3 \int_{-\sqrt{9-x^2}}^0 e^{x^2+y^2} \mathrm{d}x \mathrm{d}y.$$

(b) Use the techniques we learnt in class to find the volume of a sphere.
Hint: Just as in the DS, begin by motivating that

$$V = \iiint_{\mathbb{R}^3} \mathrm{d}x \mathrm{d}y \mathrm{d}z f(x, y, z), \qquad \text{where } f(x, y, z) = \begin{cases} 1, & x^2 + y^2 + z^2 \le R^2 \\ 0, & x^2 + y^2 + z^2 > R^2 \end{cases},$$

and then shift to a coordinate system that respects the symmetry of the problem. **[3]**

(c) Use a similar technique to calculate the **surface area** of a sphere. **[2]**