Assignment 12: Vector Calculus

Suggested Submission: May 07, 2020 (Thursday) **Marks: 15**

(a) Consider this scalar field representing, say, the depth of a lake. Find its gradient. **[2]**

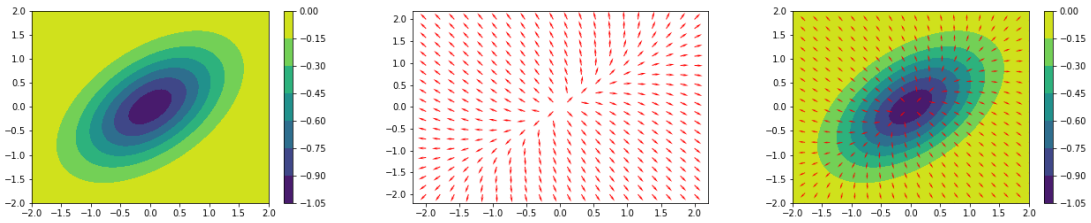
$$D(x,y) = -e^{-x^2+xy-y^2}.$$

(b) **Computation:** **[5]**

(i) Let us now try to plot this scalar function using a contour plot. Read up about Contour Plots from here. Your end result should look something like Figure (1a).

(ii) Separately, plot the gradient (which is a vector field which takes different values at different points) of this function. Read up on plotting 2D vector fields from here. You should get something that looks like Figure (1b).

(iii) Lastly, plot both of them together on the same graph, so that you can see the direction of steepest descent at every point. You can find information of how to do this in this very helpful StackOverflow answer. You should get something like Figure (1c).



(a) A contour plot of $D(x,y)$ (b) A vector plot of ∇D (c) A combined plot

Figure 1: Three plots that you need to provide in your answer.

(c) It should be clear that ∇D is a vector field. Calculate its Divergence and Curl. **[4]**

(d) Using the definitions of the Divergence and Curl, show that for *any* vector field $\mathbf{F}$, **[2]**

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0.$$

(e) Using the definitions of the Gradient and Curl, show that for *any* scalar field ϕ, **[2]**

$$\nabla \times \big(\nabla \phi\big) = 0.$$