Assignment 2:
An Introduction to Vector Spaces

Due: February 13, 2020 (Thursday) **Marks: 20**

1 Affine Spaces

Here are some theorems in Euclidean plane geometry. Discuss whether each is a valid statement in *affine* plane geometry. **[10]**

(a) The medians of a triangle meet at a point which is 2/3 of the way from each vertex to the midpoint of the opposite side.

(b) The angle bisectors of an isosceles triangle are equal in length.

(c) The diagonals of a rhombus are perpendicular.

(d) The diagonals of a parallelogram bisect each other.

(e) Let PQR and $P'Q'R'$ be two triangles such that the lines PQ and $P'Q'$ are parallel, QR and $Q'R'$ are parallel, and PR and $P'R'$ are parallel. Then the three lines PP', QQ', and RR' are either parallel or concurrent.

2 Vector Spaces

Consider the set of all real polynomials of degree less than or equal to a non-negative number n, $P_n(x)$. An arbitrary element of this space could be, for example:

$$p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n,$$

where the numbers $a_0, a_1, \ldots, a_n$ are any real numbers, and any of the a_is could be zero. Answer the following questions in detail, with sufficient justification.

(a) If the rule of addition is defined in the usual way, does this set constitute a vector space? Take every postulate and explain why (or why not) it is satisfied by $P_n(x)$. **[5]**

(b) Find a possible set of basis vectors for $P_n(x)$. (That is, find the smallest set of linearly independent elements required to completely describe an *arbitrary* element of $P_n(x)$. **[3]**

(c) What is the *dimension* of this space? Explain. **[2]**