Assignment 6: Programming

Due: March 19, 2020 (Thursday) **Marks: 15**

1 Non-linear drag

Consider a more realistic model for an object falling through a viscous material than the one discussed in your exam: suppose the damping force is proportional to the *square* of the velocity (and opposite to its direction[1]):

$$\mathbf{F}_d = -\frac{1}{2}\rho C_d A v^2 \hat{\mathbf{v}}.$$

where $\hat{\mathbf{v}}$ is the unit vector in the direction of the velocity, A is the cross-sectional area of the object, C_d is a quantity called the drag coefficient which is different for different shapes, and ρ is the density of the material that the object is falling through.

(a) Write a simple code to simulate a sphere of radius 1 cm and mass 1 g dropped from rest, falling through glycerine. (Look online for any values you require.) **[3]**

(b) Imagine you are dropping three objects (a cube, a sphere, and a cone) of the **same mass** m, and **same cross-sectional area** A. On a single graph, plot all three of their trajectories. You may assume any initial conditions you wish. **[2]**

2 The one-body problem

We have discussed the Kepler problem (the Earth going around the sun) in great detail, the code I used in the DS is available on the Google Drive. Let's now try to study this problem in a little more detail.

Let's begin by modifying the code a bit:

(a) Write a program which integrates Newton's equation of motion for a single planet under the influence of the gravitation of the sun (which you can assume not to be moving) from time $t = 0$ to some time T, using a time step dt. Consider a generalised gravitational force,

$$F = -\frac{GMm}{r^\beta},$$

[1] This condition makes it *slightly* harder to model than the simpler case of linear drag. Think carefully!

where $\beta = 2$ corresponds to "normal" gravity). Record the position, velocity, kinetic energy, potential energy and total energy as functions of time.[2] **[3]**

(b) Determine a reasonable value for the time step dt. Begin by guessing, and justify your guess. For the Earth, and using $\beta = 2$, optimise dt by using the requirement of total energy conservation. To this end, calculate the energy change over one orbit and plot it as a function of dt. **[3]**

(c) Try to verify Keplers's third law for all planets in the Solar System. **[4]**

[2] This is pretty much already done in the code provided to you, you'll just need to edit it to include the quantities I mentioned as *parameters*.