Assignment 8: Coupled Oscillators and Normal Modes

Due: April 9, 2020 (Thursday) **Marks: 15**

Theoretical physics is not just doing calculations. It's setting up the problem so that any fool could do the calculation.

– Phil Anderson (1923 – 2020)

1 Four Masses and Five Springs

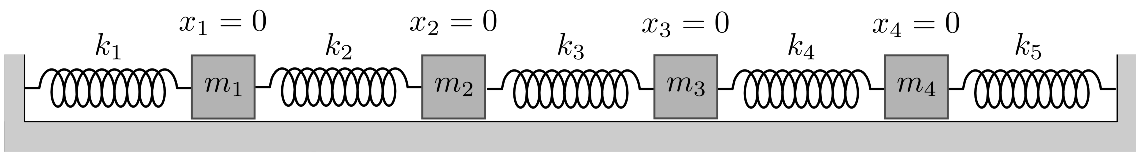

Figure 1: Four masses are connected by five springs on a frictionless surface.

(a) Consider the situation given in Figure (1). For this general case, find the equations of motion for each mass, and write them out in matrix form as $\underline{\underline{M}} \cdot \ddot{\underline{X}} = -\underline{\underline{K}} \cdot \underline{X}$, where $\underline{\underline{M}}$ and $\underline{\underline{K}}$ are two matrices you must find. **[2]**

(b) Now consider the simpler case when $m_1 = m_2 = m_3 = m_4 = m$, and $k_1 = k_2 = k_3 = k_4 = k_5 = k$. Choosing $\omega_0 = k/m = 1$, solve this on Sympy to find (i) the normal mode frequencies, and (ii) the normal modes. **[3]**

2 Two Masses and Two Springs

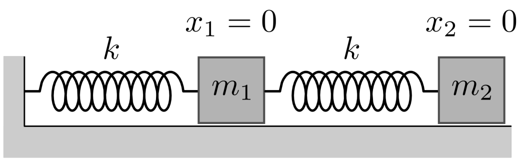

Figure 2: Two masses are connected by two springs on a frictionless surface.

Solve the situation given in Figure (2) completely (by hand) and find the general solution if initially both blocks at rest, and x_2 is displaced by 1 unit (x_1 starts from equilibrium). **[10]**

Hint: The general solution is a linear superposition of the normal modes of the system.