DS 6:
Linear Transformations

Philip Cherian

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1 Linear Transformations

(a) Show that the two conditions for a transformation L to be linear

(i) $L(u+v) = L(u) + L(v)$, for two vectors u and v,

(ii) $L(cv) = cL(v)$, for a vector v and a scalar c.

is equivalent to the single condition $L(ru+sv) = rL(u) + sL(v)$, where r and s are scalars.

Hint: You need to do this in two parts. First show (i, ii) $\implies$ (iii), and that (iii) $\implies$ (i,ii).

(b) Consider the vector space $P_n(x)$ of polynomials of order up to n. Suppose L is a linear transformation from P_2 to P_3 such that

$$L(1) = 4, \quad L(x) = x^3, \quad L(x^2) = x - 1.$$

Find:

(i) $L(1+t+2t^2)$

(ii) All values a, b, c, such $L(a+bt+ct^2) = 1+3t+2t^3$.

(c) Solve Exercise (1.4) from the textbook *A Course in Mathematics for Stuents of Physics: Volume 1* by Paul Bamberg and Shlomo Sternberg, (Pg. 47).

2 Kernel and Image

(a) Find the kernel and image for the following transformations, and draw them out. In each case, show that

$$\dim(\ker(F) + \dim(\operatorname{Im}(F) = \dim(V).$$

(i) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$, such that

$$F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+2y \\ -6x-4y \end{pmatrix}.$$

(ii) Let $F : \mathbb{R}^3 \to \mathbb{R}^3$, such that

$$F\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+y+8z \\ y+2z \\ x+y+5z \end{pmatrix}.$$

(b) For each of the cases above, find the matrix that represents the transformation F.

(c) Prove that the kernel of a linear transformation is a vector (sub)space.