DS3:
Relativity and the Lorentz Transformations

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10th September, 2018

1 Relativistic Aberration of Angles

Consider a rod inclined at an angle $\tan\theta$ in the $xy-$plane in its rest frame. When observed in a frame in which this rod is found to be moving at a velocity v in the x direction, what is the angle θ' that the rod makes with the $x'y'-$plane?

2 Whither Newton?

1. Let us call the four-vector that you discovered in Dr. Phookun's last assignment **U**.
2. Let us now define a new four-vector **A**, which is given by

$$\mathbf{A} = \frac{\mathrm{d}\mathbf{U}}{\mathrm{d}\tau}$$

   where τ is the proper time as earlier defined. What is A a generalisation of? Take the non-relativistic limit.

3. Prove that $U_\mu A^\mu = 0$.
4. Let's go back to **U**: multiplying it with the mass of the particle m, let us define another four-vector **P**:

$$P^\mu = mU^\mu$$

   What is the norm of this four-vector?

5. Let us write this vector as

$$P^{\mu} = \begin{pmatrix} \frac{E}{c} \\ \mathbf{p} \end{pmatrix}$$

Write down E explicitly. Now, take the non-relativistic limit (i.e. saying $v \ll c$), ignoring all but the first two terms. Interpret these two terms.

3 Partial Derivatives

As an exercise, consider the transformations that take you from Cartesian to Polar coordinates.

$$\begin{aligned} r^2 &= x^2 + y^2 \\ \theta &= \tan^{-1}\left(\frac{y}{x}\right) \\ x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

1. **Bonus:** Can you find the transformation matrix for this transformation?
2. Calculate the following partial derivatives in terms of x and y and their partial derivatives:

$$\frac{\partial}{\partial r} \quad \text{and} \quad \frac{\partial}{\partial \theta}$$

3. Calculate the following partial derivatives in terms of r and θ and their derivatives:

$$\frac{\partial}{\partial x} \quad \text{and} \quad \frac{\partial}{\partial y}$$

4 Not for the faint-hearted: A rotating cylinder

Consider the following situation: you are told that there is a cylinder that – in its rest frame S' – is rotating along the x' axis (i.e. in the $y'z'-$plane with a constant angular velocity ω').

Consider observing this cylinder from a frame S in which it appears to be moving with a constant velocity in the $x-$direction. We will try to see what we observe happen to this cylinder, as depicted in the below figure.

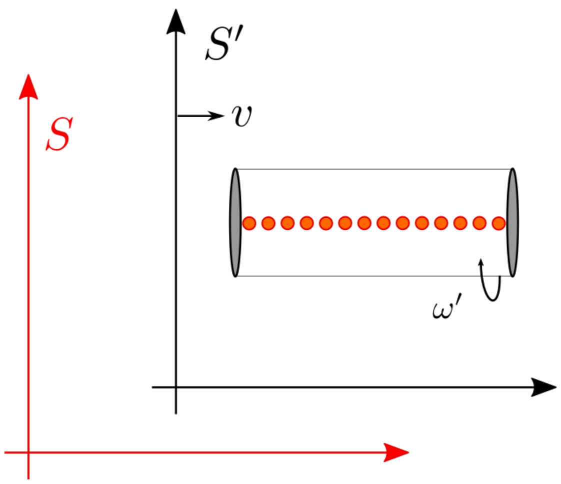

Figure 4.1: A rotating cylinder in its rest frame S' is viewed from S.

1. An observer in S' knows that the cylinder turns with a constant angular velocity ω'. She can thus create a clock using any of the points marked in the figure. When the point completes a full circle, she could define a unit of time.

   Now imagine that she does this using an arbitrary point on the cylinder (say the left-most orange dot), and a time T' is measured. What will be the time T measured by you in S (for the same point)?

2. Now consider two points separated in space in S'. The observer in S' decides to synchronise all clocks in her frame such that they 'tick' at the same time. In other words, to her, all the clock's hands are parallel at any moment in time.

   Will you – the observer in S – see all the hands move parallel to each other? Why?

3. As a result of the last question, you must now be able to visualise how the cylinder looks differently to you: the cylinder appears **twisted**. Calculate the **torsion per unit length** as measured in S.

4. Would you now expect any new stresses and strains in the cylinder? Why?

5. **Bonus:** Find the equation of the orange line in the above figure, as observed in S.

6. Discuss what profound implications this might have to the the general well-being of cylinders.