DS6 & DS7:
Gauss's Law and the Principle of Superposition

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1 Gauss's Law

1. Calculate the electric field (**everywhere**) due a solid sphere of volume charge density ρ and radius R.
2. Calculate the electric field (**everywhere**) due to a thin spherical shell of surface charge density σ, and radius R using Gauss's law.
3. Calculate the electric field (**everywhere**) due to a ***thick*** spherical shell of inner radius R_1 and outer radius R_2, and volume charge density ρ using Gauss's law.

2 The Principle of Superposition

1. Calculate the electric field due to a **ring of charge** of linear charge density λ and radius R at a distance z from its centre along its axis.
2. Calculate the electric field due to a **disc of charge** of surface charge density σ and radius R at a distance z from its centre along its axis.
3. Calculate the electric field due to a **cylinder of charge** of thickness l, volume charge density ρ and radius R at a distance z from its centre along its axis.
4. Calculate the electric field of an **infinite plate of charge** of thickness l, and volume charge density ρ at any distance z above it, using the previous answers.
5. Assume you know the electric field everywhere due to a solid sphere of charge that you found using Gauss's law in Question 1 of Part 1. Try to combine two solid spheres

using the Principle of Superposition and calculate the Electric Field **everywhere** due to a **thick shell** of inner radius R_1 and outer radius R_2 and volume charge density ρ.

6. Assume you know the electric field everywhere due to a thin shell that you found using Gauss's law in Question 2 of Part 1. Now, integrate this using the principle of superposition to find the Electric Field **everywhere** due to a **thick shell** of inner radius R_1 and outer radius R_2 and volume charge density ρ.

3 Index Notation: In far more depth than required

If you want to do these questions but cannot, meet me during office hours.

All questions assume we're in THREE DIMENSIONS.

1. Suppose you have an orthogonal basis of unit vectors, given by $\{\hat{e}_n\}$, $n = 1,2,3$. Show that

$$\hat{e}^i \hat{e}_j = \delta^i_j \tag{3.1}$$

where δ^i_j is the **Kronecker delta** (look it up if you don't know what it is).

2. You are given the following equation. What does K represent, physically?

$$K^i = cF^{\mu i}\delta^0_\mu \tag{3.2}$$

where the Greek indices $\mu = 0,1,2,3$ are space-time indices, the Latin indices $i = 1,2,3$ are spatial indices, and $F^{\mu\nu}$ is the Electromagnetic Field Tensor.[1]

3. Find δ^i_i

4. For the same basis vectors defined in the first question, consider the following **definition** for an object ϵ_{ijk}

$$\hat{e}_i \times \hat{e}_j = \epsilon_{ijk}\,\hat{e}_k \tag{3.3}$$

Calculate the components of ϵ_{ijk}. Show that the following statements are true:

a) $\epsilon_{ijk} = 1$ (all indices being distinct and cyclic, the value is 1)

b) $\epsilon_{ijk} = \epsilon_{jki} = \epsilon_{kij} = 1$ (cyclic permutations preserve the value)

c) $\epsilon_{ijk} = -\epsilon_{jik} = -\epsilon_{kji}$ (flipping a single pair of indices inverts the sign of the value)

d) $\epsilon_{iij} = \epsilon_{ijj} = \epsilon_{iji} = \epsilon_{iik} \ldots$ etc. $= 0$ (when any two indices are the same, value is 0)

[1] I haven't told you what c is, but you should know by now.

5. Show that

$$(\vec{a} \times \vec{b})_k = \epsilon_{ijk} a_i b_j \tag{3.4}$$