DS8 & DS9:
Magnetostatics and the Principle of Superposition

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1 Densities

Here is a little 'dictionary', courtesy of Griffiths:

For charge distributions:

$$\sum_{i=1}^{n}(\quad)q_i = \int_{\text{line}}(\quad)\lambda \mathrm{d}l = \int_{\text{surface}}(\quad)\sigma \mathrm{d}S = \int_{\text{volume}}(\quad)\rho \mathrm{d}V$$

For current distributions:

$$\sum_{i=1}^{n}(\quad)q_i\mathbf{v_i} = \int_{\text{line}}(\quad)\mathbf{I}\mathrm{d}l = \int_{\text{surface}}(\quad)\mathbf{K}\mathrm{d}S = \int_{\text{volume}}(\quad)\mathbf{J}\mathrm{d}V$$

This is slightly less intuitive for current densities than it is for charge densities, so we'll go over some simple exercises.

1. A current I is distributed over a cylindrical wire of radius a.

   a) Suppose now that the current is distributed over the surface of the wire. Find the surface current density K.

   b) If the current is distributed uniformly in the bulk, find the volume current density J.

c) Now suppose instead that the volume current density in the wire is proportional to the distance from the axis. What is J?

d) Finally, suppose the volume current density is inversely proportional to the distance from the axis. What is J?

2. Imagine a vinyl record carrying a uniform charge density σ. Suppose it is rotated at a constant angular velocity ω. What is the surface current density at a distance r from the centre?

3. Consider a sphere carrying uniform charge density σ. Suppose *it* is rotated at a constant angular velocity ω. What is the surface current density at different points on the sphere?

2 The Biot-Savart Law

1. The following objects are rotated along their axes with an angular velocity ω. Find the magnetic field at a distance z on their axis of rotation:

   a) A ring of radius r with linear charge density λ.

   b) A disc of radius R with surface charge density σ.

   c) A spherical shell of radius R with surface charge density σ.

   d) A solid sphere of radius R with volume charge density ρ.

2. Consider a plane loop carrying a steady current I, due to which we want to find the field on the plane. Without loss of generality, we could choose that point to be the origin of our coordinate system (Note that the point may be inside or outside the loop). The shape of the loop is given – in polar coordinates – by $r(\theta)$.

   a) Beginning with the Biot-Savart law, show that the magnitude of the magnetic field is given by

   $$B = \frac{\mu_0 I}{4\pi} \oint \frac{\mathrm{d}\theta}{r}$$

   b) What is $r(\theta)$ for a circular loop? Test the above formula by calculating the field at the centre of such a loop.

   c) Conic sections are defined by a 'semilatus rectum' p and an eccentricity e. ($e = 0$ for a circle, $0 < e < 1$ for an ellipse, $e = 1$ for a parabola):

   $$r(\theta) = \frac{p}{1 + e\cos\theta}$$

   How does the magnitude of the magnetic field depend on the eccentricity?

3 MAGNETOSTATICS

1. Calculate the magnetic field at the centre of a square loop of side l.
2. Calculate the magnetic field at the centre of a regular $n-$sided polygon. Take the limit $n \to \infty$. What do you see?
3. Imagine you have two infinite wires of linear charge densities λ, moving with a velocity v, separated by a distance d.
   a) Calculate the electrical force between them.
   b) Calculate the magnetic force between them.
   c) What would v have to be so that these two forces balance out?
4. Imagine a copper wire 1 mm in diameter, carrying a current of 1 A.
   a) Calculate the density ρ of charge carriers in a piece of copper, assuming each atom contributes one electron.
   b) Calculate the average electron velocity in the wire.
   c) Calculate the force of attraction between two such wires 1 cm away. Calculate the electric force between them.
5. We will now try to answer why currents within a single wire do not contract into a tiny concentrated stream along the axis. Imagine the positive charges ρ_+ are nailed down, and the negative charges ρ_- move at some speed v with respect to them.
   a) Imagine a single charge element within the wire. What is the condition that must be met by the Electric and Magnetic fields that act on the element so that an equilibrium might be met?
   b) Calculate the Electric and Magnetic fields in the material at equilibrium.
   c) Use this to show that

   $$\rho_+ + \rho_- = \rho_- \left(\frac{v^2}{c^2}\right)$$

   and consequently that

   $$\rho_- = -\gamma^2 \rho_+$$

   d) How does the charge distribution look, given that the wire is overall neutral?