Computational Project:
The Relaxation Method

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Two-dimensional Electrostatics

Your project is as follows: you will need to write a program in Python to solve Laplace's equation in two dimensions for different boundary conditions. In order to do this, you will need to use something called the *relaxation method*, or one of its variants.

Dr. Phookun has managed to come up with seven boundary conditions, so you can work in groups of two. You can find these boundary conditions in the figure below. Depending on the boundary conditions you'll have to choose either Cartesian or polar coordinates.

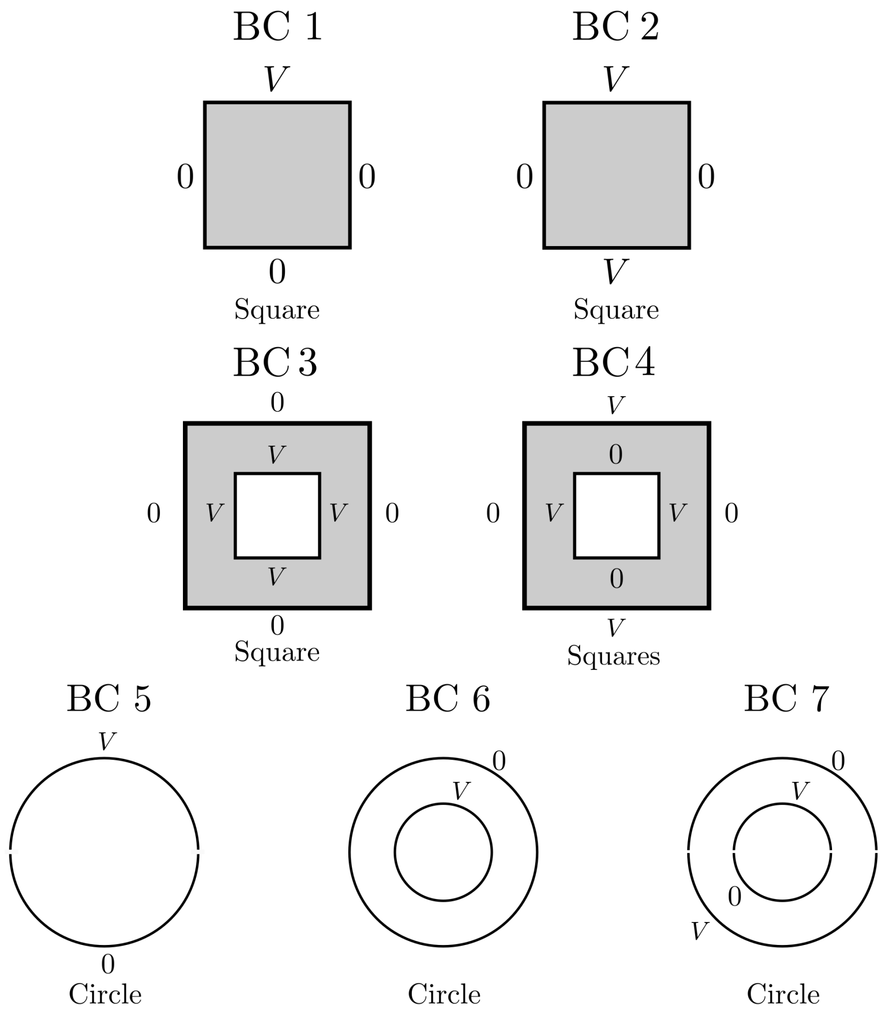
After arriving at the solution, plot a series of equipotential surfaces to show what you have got. Please make sure your answer makes sense.

The Relaxation Method

A quick introduction to the relaxation method can be found on Wikipedia here, but the basic idea is really quite simple: to understand it, we use a curious property of Laplace's Equation, which is that the value at any point (x, y) is the average of the values at all equidistant neighbouring points.[1] In other words, $V(x, y) = \frac{1}{4}\left(V(x-h, y) + V(x+h, y) + V(x, y-h) + V(x, y+h)\right)$

Begin by defining a grid, and setting the boundary conditions (remember, the values at the conductors *never* change). Then, loop over the entire grid, and to every point assign the potential a value that is the average of the potential at the four points around it. After a sufficiently large (how will you find out how large?) number of iterations, the potential $V(x, y)$ will be the solution for the problem (by the uniqueness of solutions of Laplace's Equation).

[1] This statement and those that follow apply for two-dimensional systems. They can all be appropriately generalised to three dimensions, though it will not be necessary for this project.



Different 2D boundary conditions: the potential needs to be found in the region enclosed by the conductors. Conductors at different potentials that seem to “meet” are in fact separated by small amounts of insulation.