Assignment 3:
Coupled Equations and Normal Modes

Due: February 11, 2022 (Friday) **Marks: 15**

Theoretical physics is not just doing calculations. It's setting up the problem so that any fool could do the calculation.

– Phil Anderson (1923 – 2020)

1 Coupled Differential Equations

Consider the system of equations $\dot{x} = -2x + y$ and $\dot{y} = x - 2y$.

(a) Find the eigenvalues and eigenvectors of the matrix M associated with these equations. **[2]**

(b) We have seen that the formal solution to this equation is $V(t) = e^{tM}V(0)$. Solve the equation explicitly by diagonalising the matrix M. You can take for initial conditions $x(0) = 1$, $y(0) = 0$. **[2]**

2 Coupled Oscillators

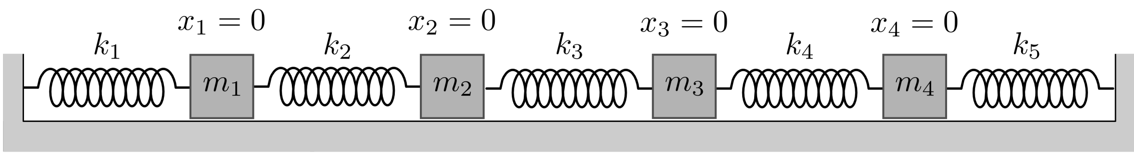

Figure 1: Four masses are connected by five springs on a frictionless surface.

(a) For the general case, given in Figure (1), find the equations of motion for each mass, and write them out in matrix form as $\underline{\underline{M}} \cdot \ddot{\underline{X}} = -\underline{\underline{K}} \cdot \underline{X}$, where $\underline{\underline{M}}$ and $\underline{\underline{K}}$ are two matrices you must find. **[2]**

(b) Solve the situation given in Figure (2) completely and find the general solution if both blocks are initially at rest, and x_2 is displaced by 1 unit (x_1 starts from equilibrium). **[7]**

(c) Plot two graphs for the solution to part (b): (i) the positions of both masses x_1 and x_2, and (ii) the movement of the normal modes of the system as a function of time. **[2]**

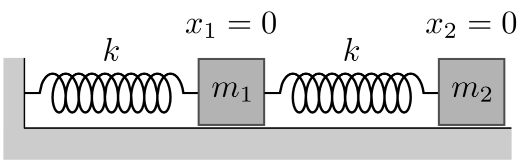

Figure 2: Two masses are connected by two springs on a frictionless surface.

3 Bonus:

(a) Solve the Simple Harmonic Oscillator in as many different ways as possible. You will get one bonus mark for every sufficiently distinct (and clearly explained!) way you find of solving the equation. And if you find a way I haven't heard of yet, you will get an even larger bonus! [**Bonus** ∞]