Assignment 10: Transmission across Thin Films

Due: April 22, 2022 (Sunday) **Marks: 15**

1 Transmission coefficient of a thin film

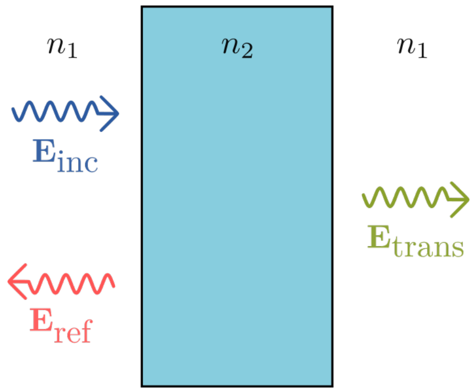

Figure 1: A thin film of refractive index n_2 is placed in a material of refractive index n_1. Thus, our problem has three regions of space, with refractive indices n_1, n_2, and n_1, respectively.

In the previous assignment, we saw that the transmission across a thin film like the one shown in Figure (1) can be found using the transmission matrix

$$\underline{\underline{M}} = \underline{\underline{D_{12}}} \cdot \underline{\underline{P_2}} \cdot \underline{\underline{D_{21}}}, \tag{1}$$

where $\underline{\underline{D}}$ denoted the transfer matrix and $\underline{\underline{P}}$ the propagation matrix found in the previous assignment. The reflection and transmission coefficients can be found from the components M_{ij} of the matrix $\underline{\underline{M}}$, since

$$t = \frac{1}{M_{11}} \qquad \text{and} \qquad r = \frac{M_{21}}{M_{11}}. \tag{2}$$

(a) Compute the transmission coefficient for the system in Figure (1), and show that it reduces to **[8]**

$$T = |t|^2 = \frac{1}{\cos^2(k_2 L_2) + \left(\frac{n^2+1}{2n}\right)^2 \sin^2(k_2 L_2)}, \tag{3}$$

where $n \equiv n_2/n_1$, and $k_2 = n_2\omega/c$.

(b) If you look at this equation for long enough, you should notice something very strange: show that there are special values of L_2, such that the transmission coefficient is identically 1! Compute these values. In other words, there are some thicknesses for which the entire wave incident on the film is transmitted! Can you guess what is happening at these thicknesses that allows for this? (You might want to do the next part of the assignment before coming back to answer this.) **[2]**

2 "Resonances"

There is a very interesting interpretation to this, a version of which you will (hopefully) see during your course on *Quantum Mechanics* next semester. We will try to explore this graphically in this part of the assignment.

(a) Plot three curves (on the same graph) of T against $x = k_2 L_2$ for $n = 1.5$, $n = 3$, and $n = 9$. You should see that T has peaks, and that these peaks become "finer" as n increases. **[2]**

(b) Now, do a Taylor Series Expansion about an arbitrary peak, and show that close to peak number p, T looks like

$$T = \frac{1}{1 + \left(\frac{1+n^2}{2n}\right)^2 (x - p\pi)^2}. \tag{4}$$

You have seen a function like this at the start of the course (where?). It is called a *Lorentzian*, generally written as

$$L(x, x_0, \gamma) = \frac{\gamma^2}{\gamma^2 + (x - x_0)^2}. \tag{5}$$

Such curves are characteristic of resonance processes, where some external driving phenomena matches the internal natural response of a system. The parameter γ determines how "fine" the peak is, and therefore how selective the resonance is to the external phenomena.[1]

For one of the peaks (say, $p = 1$), recreate the same plot as before, with the transmission coefficient T as well as the approximate Lorentzian that we have just computed to see over what range the two curves overlap. **[3]**

[1] In optics, it is related to a quantity called the "finesse".