DS 6: Irreversibility, Heat Flow, and Coupled Systems

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1 Heat-flow

(a) Consider a cylinder of length L and cross-sectional area A, divided into two equal-volume chambers by a piston that is fixed externally at the midpoint of the cylinder. One half of the cylinder contains N moles of a monoatomic ideal gas at a temperature T_0. Within this chamber, the piston is held by a spring of constant k, whose unstretched length is $L/2$, so that at equilibrium the spring initially exerts no force on the piston. The other chamber is completely evacuated, and the walls and the piston are adiabatic.

Suppose the piston is suddenly released. You will now need to find the final volume and temperature of the gas at equilibrium.

(i) Write down the equation of energy conservation for this system.

(ii) Write down the equation of mechanical equilibrium. **Hint:** Why doesn't the gas expand into the entire cylinder?

(iii) Using the equation of state at equilibrium, find a relationship between the distance moved by the piston and the initial temperature of the gas T_0.

(iv) Use the above result to find the final volume and temperature of the system.

(b) Consider a series of $N+1$ large vats of water that have temperatures $T_0, T_1, \ldots T_N$, with $T_i > T_{i-1}$. A small body with heat capacity C and constant volume (independent of temperature) is initially in thermal equilibrium with T_0. The body is removed from this vat, and immersed in the vat of temperature T_1 until it is at equilibrium. This process is repeated until, after N steps, the body is at equilibrium with the vat at T_N. The sequence is then reversed, until the body is once again in the initial vat, at temperature T_0. Assuming the ratios of the temperatures of successive vats to be a constant, in other words, assuming

$$\frac{T_i}{T_{i-1}} = \left(\frac{T_N}{T_0}\right)^{1/N}, \tag{1}$$

and neglect any change in the temperature of the vats.

(i) Calculate the total entropy as the body is successively taken "up" the sequence (from T_0 to T_N).

(ii) Calculate the total entropy as the body is successively taken "down" the sequence (from T_N to T_0).

(iii) Find the total change in entropy in the sum of both sequences.

(iv) Find the leading non-trivial result of these results as $N \to \infty$ (but keeping T_0 and T_N constant).