Problem Set 1

Problems from Callen (Chapters 2 & 3)

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Problem 2.27 A particular system obeys the relation

$$u = Av^{-2}\exp(s/R). \tag{1}$$

N moles of this substance, initially at temperature T_0 and pressure P_0, are expanded isentropically until the pressure is halved. What is the final temperature?

Problem 2.6-3 Two particular systems have the following equations of state:

$$\frac{1}{T^{(1)}} = \frac{3}{2}R\frac{N^{(1)}}{U^{(1)}}, \qquad \text{add} \qquad \frac{1}{T^{(2)}} = \frac{5}{2}R\frac{N^{(2)}}{U^{(2)}}, \tag{2}$$

where R is the gas constant. The mole number of the first system is $N^{(1)} = 2$, and that of the second $N^{(2)} = 3$. The two systems are separated by a diathermal wall, and the total energy of the composite system is $2.5 \times 10^3 J$. What is the internal energy of each system at equilibrium?

Problem 3.3-2 It is found that a particular system obeys the relations

$$U = PV \qquad \text{and} \qquad P = BT^2, \tag{3}$$

where B is a constant. Find the (entropic) fundamental equation of this system.

Problem 3.4-5 In particular engine, an ideal gas is compressed in the initial stroke of a piston. Measurements of the instantaneous temperature, carried out during the compression, reveal that the temperature increases according to

$$T = \left(\frac{V}{V_0}\right)^{\eta} T_0, \tag{4}$$

where T_0 and V_0 are the initial temperature and volume, and η is a constant. The gas is compressed to the volume V_1, where $V_1 < V_0$. Assume the gas to be a monoatomic ideal gas, and assume the process to be quasi-static.

(a) Calculate the work done W on the gas.

(b) Calculate the change in energy ΔU of the gas.

(c) Calculate the heat transfer Q to the gas (through the cylinder walls) by using the results of (a) or (b).

(d) Calculate the heat transfer *directly*, by integrating $đQ = T\,dS$.

(e) From the result of (c) or (d), for what value of η is $Q = 0$? Show that for this value of η, the locus traversed coincides with an adiabat (as calculated in Problem 3.4-2).

Problem 3.4-9 A tank has a volume of 0.1 m^3, and is filled He gas at a pressure of 5MPa. A second tank has a volume of 0.15m^3, and is filled with He gas at a pressure of 6MPa. A valve connecting the two tanks is opened. Assuming He to be a monoatomic ideal gas, and the walls of the tanks to be adiabatic and rigid, find the final pressure of the system. **Hint:** Note that the internal energy is constant.

Problem 3.5-4 Repeat parts (a), (b), and (c) of **Problem 3.4-5**, assuming that $\eta = -1/2$, and that the gas is an ideal van der Waals fluid. Show that your results for ΔU and for W (and hence for Q) reduce to the results of Problem 3.4-5 (for $\eta = -1/2$) as the van der Waals constants a and b go to zero, and $c_v = (3/2)Nk_B$.

Hint: Recall that for a van der Waals gas

$$u = cRT + \frac{a}{v}, \qquad \text{and} \qquad P = \frac{RT}{v-b} - \frac{a}{v^2}. \tag{5}$$

Problem 3.9-6 A simple fundamental equation that exhibits some of the qualitative properties of typical crystaline solids is

$$u = Ae^{b(v-v_0)^2} s^{4/3} e^{s/3R}, \tag{6}$$

where A, b, and v_0 are positive constants.

(a) Show that the system satisfies the Nernst theorem.

(b) Show that c_V is proportional to T^3 at low temperature. This is commonly observed (and was explained by P. Debye by a statistical mechanical analysis, which will be developed in a later course).

(c) Show that $c_V \to 3k_B$ at high temperatures. This is the "equipartition value", which is observed and will again be demonstrated in a later course on statistical mechanics.

(d) Show that for zero pressure, the coefficient of thermal expansion vanishes in this model – a result that is *incorrect*. **Hint:** Calculate the value of v at $P = 0$.