Problem Set 2

Problems from Callen (Chapter 3)

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Problem 3.4-2 Show that the relation between the volume and the pressure of a monoatomic ideal gas undergoing a quasi-static adiabatic compression $đQ = T\mathrm{d}S = 0$, i.e. $S =$ constant) is

$$P v^{5/3} = \left(P_0 v_0^{5/3} e^{-2s_0/3R}\right) e^{2s/3R}. \tag{1}$$

Problem 3.4-3 Two moles of a monoatomic ideal gas are at a temperature of $0°C$ and a volume of 45 litres. The gas is expanded adiabatically and quasi-statically until its temperature falls to $-50°C$. What are its initial and final pressures, and its final volume?

Problem 3.4-4 By carrying out the integral $\int P\mathrm{d}V$, compute the work done by the gas in **Problem 3.4-3**. Also compute the initial and final energies, and corroborate that the difference in these energies is the work done.

Problem 3.4-6 Find the three equations of state of the "simple ideal gas". Show that these equations of state satisfy the Euler relation.

Hint: A simple ideal gas is one that satisfies

$$S = Ns_0 + NR\ln\left[\left(\frac{U}{U_0}\right)^c\left(\frac{V}{V_0}\right)\left(\frac{N}{N_0}\right)^{-(c+1)}\right], \qquad \text{where} \quad s_0 = (c+1)R - \left(\frac{\mu}{T}\right)_0 \tag{2}$$

Problem 3.4-8 If a monoatomic ideal gas is permitted to expand into an evacuated region, thereby increasing its volume from V to λV, and if the walls are rigid and adiabatic, what is the ratio of the initial and final pressures? What is the ratio of the initial and final temperatures? What is the difference of the initial and final entropies?

Problem 3.4-11 Show that the pressure of a multicomponent simple ideal gas can be written as the sum of "partial pressures" P_j, where $P_j \equiv N_jRT/V$. These "partial pressures" are purely formal quantities not subject to experimental observation. (From the mechanistic viewpoint of kinetic theory, the partial pressure P_i is the contribution to the the total pressure that results from bombardment of the wall by molecules of species i – a distinction that can be made only when the molecules are non-interacting, as in an ideal gas.)

Problem 3.4-13 An impermeable, diathermal, and rigid partition divides a container into two subvolumes, each of volume V. The subvolumes contain, respectively, one mole of H_2, and three moles of *Ne*. The system is maintained at a constant temperature T. The partition is suddenly made permeable to H_2, but not to *Ne*, and equilibrium is allowed to reestablish. Find the mole numbers are the pressures.

Problem 3.9-1 Show that for the multicomponent simple ideal gas

$$c_V = \bar{c}R, \qquad \alpha = \frac{1}{T}, \qquad \kappa_T = \frac{1}{P}, \qquad \kappa_S = \frac{\bar{c}}{\bar{c}+1}\frac{1}{P}, \qquad \text{and} \quad c_P = (\bar{c}+1)R \tag{3}$$

where $\bar{c} = \sum_j c_j N_j / N$.