Computational Project: Random Walks

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April 6, 2024

In this computational project, you will simulate a *random walk*, a very important idea in physics that can be used to describe Brownian motion, diffusion, polymer physics, and even has applications in more complicated domains like quantum field theory. It also finds use in a wide variety of other fields, from biology to financial economics.

1 The one-dimensional random walk

Let us first consider an N-step random walk on a one-dimensional lattice. The walker starts off at the origin ($x = 0$) and at every step, moves to one of its nearest neighbours with a probability p and $(1 - p)$ respectively. For the simplest random walk, you can take $p = 1/2$, meaning that both of the nearest neighbours are equally likely. In what follows, you can make this assumption.

(a) Write a function `walk_1D` that simulates a one-dimensional random walk. This function should accept the two parameters given below, and should return an array of coordinates of each step.

- `p`: the probability of the walker moving "rightwards",
- `n_steps`: the number of steps of the random walk,

(b) Simulate a large number of walks (say, `n_walks=1000` or higher) for $p = 1/2$ and record the trajectory of each random walker, storing them in an array.

(i) Plot a histogram of the final positions of all the walkers. Show that it is a Gaussian with mean $\mu = 0$ and standard deviation $\sigma = \sqrt{\texttt{n\_steps}}$.

(ii) Using the data from the last part, compute $R_d(n)$, the *end-to-end* distance: the distance between the first point and the point reached after n steps. Plot this distance as a function of n going from `0` to `n_steps`. Remember: this average has to be taken over *multiple walks*.

(c) Now, take any introductory text book on random walks and show *theoretically* that

$$\langle R_d^2(n) \rangle = n. \tag{1}$$

Compare your computational result with the theoretical one. Do they match? What if you vary p?

2 The two-dimensional random walk

Generalise the above problem to an N-step random walk on a two-dimensional grid. In such a walk, the walker starts off at the origin and, at every time step, moves a fixed amount in *either* the x or y directions. Thus, at every time-step, she randomly chooses between one of four possibilities. This is then repeated N times. Repeat all the sub-parts of the previous question. In part (b)(i), you will need to plot two histograms, one for each dimension. In part (b)(ii), $\langle R_d^2(n) \rangle$ must be the modulus of the *vector* sum.