DS 10:
Algebraic Treatment of Quantum Problems

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The Harmonic Oscillator

(a) Calculate the expectation value $\langle x \rangle$ and $\langle p \rangle$ in an energy eigenstate of the Harmonic Oscillator $|n\rangle$.

(b) Calculate Δx and Δp, and show that the uncertainty principle is satisfied.

(c) Solve the equations for the Heisenberg operators $\hat{x}(t)$ and $\hat{p}(t)$, and show that they resemble closely the classical solutions to the equations of motion.

(d) Find the probability of being outside the classically forbidden region in a state $|n\rangle$. How does this change as $n \to \infty$.

(e) Consider the following problem:

$$V(x) = \begin{cases} \infty, & x < 0 \\ \frac{1}{2} m\omega^2 x^2, & \text{otherwise.} \end{cases} \tag{1}$$

What are the energy eigenvalues and their corresponding wavefunctions?

Angular Momentum

(a) Find the wavefunction for the state $|l, l\rangle$, i.e. $Y_l^l(\theta, \varphi)$.

(b) Use this solution to find the other eigenfunctions $Y_l^m(\theta, \varphi)$