DS 13:
The Hydrogen Atom

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Quantisation of the Hydrogen Atom

Begin with the equation given in class for the function $F(\rho)$

$$\frac{\mathrm{d}^2F}{\mathrm{d}\rho^2}+\frac{2(l+1-\rho)}{\rho}\frac{\mathrm{d}F}{\mathrm{d}\rho}+\left(\frac{\zeta-2(l+1)}{\rho}F\right)=0$$

(a) Is the point $\rho=0$ a regular singular point?

(b) Assume a power series solution $\sum a_n\rho^n$ and determine the recursion relation between the coefficients.

(c) Examine the coefficients for large n. Show that

$$a_{n+1}\sim\frac{2}{n+1}a_n.$$

(d) Show that this solution (and thus $u(\rho)$) blows up as $\rho\to\infty$.

(e) What condition needs to be satisfied by ζ so that this is avoided? Show that this implies that

$$E=-\frac{\hbar^2\kappa^2}{2m}=-\frac{me^4}{8\pi^2\epsilon_0\hbar^2\zeta^2}=\frac{E_1}{n^2}.$$

The Quantum Two-Body Problem

Consider the following Hamiltonian:

$$\frac{\mathbf{P}_1^2}{2m_1}+\frac{\mathbf{P}_2^2}{2m_2}+V(|\mathbf{r}_1-\mathbf{r}_2|)$$

(a) Write out the centre of mass coordinates R and P for this system. Then write out the *relative* coordinates r and p.

(b) Show that $\left[R_i,P_j\right]=i\hbar\delta_{ij}$, and $\left[r_i,p_j\right]=i\hbar\delta_{ij}$. Show that the cross commutators are zero.