DS 6: Midterm Practice Questions

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1 Gases

(a) Compute the partition function Z for a classical ideal gas of N particles in 3 dimensions.

(i) Use Z to compute the thermodynamic free energy F and the average energy $\langle E \rangle$.

(ii) Use F to compute the entropy S, and pressure P of the system.

(iii) Show that for a (monoatomic) classical ideal gas, $C_V = (3/2)N$.

(iv) Compute the adiabatic coefficient γ for a classical ideal gas in two ways: first, using the entropy, and second using the definition of $\gamma = C_P/C_V$, and the relation $C_P - C_V = N$ in our units.

(b) Repeat the previous question for an ultra-relativistic ideal gas where the energy of each particle is given by

$$E(p) = c\sqrt{p_x^2 + p_y^2 + p_z^2} = cp. \tag{1}$$

(c) Consider a classical gas of particles in a harmonic trap such that all the particles experience a potential $V(r) = \frac{1}{2}m\omega^2 r^2$. Compute the entropy of this system. Show that the average internal energy of this system is $\langle E \rangle = 3NT$.

(d) Prove the principle of equipartition of energy.

2 Lattices

(a) Consider a lattice of distinguishable harmonic oscillators. Compute the partition function for this system, and show that the average energy is $\langle E \rangle = 3NT$.

(b) Now consider placing the same lattice of harmonic oscillators in a gravitational potential. Compute the new partition function. Which thermodynamic quantities remain the same? Which change?

(c) Consider a lattice of particles which can exist in one of three states with energies $-\epsilon$, 0, and ϵ.

(i) Compute $\langle E \rangle$ and C_V of this system as a function of temperature.

(ii) Do a Taylor series approximation of $\langle E \rangle$ and C_V and find which powers of T they vary with as $T \to 0$ and $T \to \infty$.

(d) Consider a lattice with particles that can exist in one of $g+1$ states. One state has energy 0, and the other g all have the same energy ϵ. Compute the partition function of this system. Find the $\langle E \rangle$ and C_V. Show that as $T \to 0$, the probability of the system being in the state with energy 0 is 1. Show that as $T \to \infty$ all states are equally probable.