DS 7:
The Harmonic Oscillator and the Diatomic Gas

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1 The quantum harmonic oscillator

(a) Write out the partition function for a 1-dimensional quantum harmonic oscillator.

(b) Calculate the average energy $\langle E(\omega)\rangle$of such an oscillator, as well as the specific heat C_V.

(c) Sketch C_V as a function of temperature. What happens to it as $T \to 0$ and $T \to \infty$? In what regime does it approach the classical result? Does this agree with your intuition? Why, or why not?

(d) Repeat this exercise for 2-dimensional and 3-dimensional quantum harmonic oscillators. What changes? What stays the same?

2 The diatomic gas in two-dimensions

In class we studied the diatomic gas in 3D. Let us try to repeat this exercise in two-dimensions.

(a) Recall what we did in the case of the 3D diatomic gas. Three separate regimes were explored: how will the physical problems that we solve in each of these regimes change?

(b) Study the 2D rigid-rotor. What do the energies look like for this system? Find the average energy and C_V of such a rotor as (i) $T \to 0$ and (ii) $T \to \infty$.

(c) Now study the 2D oscillator in the same regime.

(d) Lastly, what happens to the 2D free particle? Does anything change from the case in 3D?