DS 10: Blackbodies in d-dimensions

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1 Blackbody radiation

(a) Much of the arguments that we used for phonons in a previous discussion session also works for *photons*, if we consider a blackbody to be a "box" with electromagnetic waves trapped in it. Identify what remains the same, and what changes.

(b) From our earlier results, we have shown that the density of states of *photons* in 3D is given by

$$g(\omega) = \left(\frac{V}{\pi^2 c^3}\right)\omega^2. \tag{1}$$

What is the density of states in $d-$dimensions?

(c) Use this density of states to compute the energy density of a blackbody, using the fact that the energy of a photon is $\epsilon(\omega) = \hbar\omega$.

(d) Show that the total number N of blackbody photons contained in a 3D box of volume V is

$$N = \frac{2\zeta(3)}{\pi^2}\left(\frac{T}{\hbar c}\right)^3 V, \tag{2}$$

where ζ is the Riemann zeta function. What is the average energy per photon? How does this result change for a $d-$dimensional box?

(e) Show that the internal energy for blackbody radiation in 3D is

$$E = \left(\frac{\pi^4}{30\,\zeta(3)}\right) NT. \tag{3}$$

Again, how does does this change in $d-$dimensions?